1. Let $R(s, t)$ denote the smallest positive integer p such that the complete graph on p vertices, whose edges are colored red or blue, there always exists either a complete subgraph on s vertices which is entirely blue, or a complete subgraph on t vertices which is entirely red. Show that $R(3, 4) = 9$.

2. Given a positive integer. Let a_1, a_2, ..., a_{n^2+1} be a sequence of $n^2 + 1$ distinct real numbers. Show that there exists a decreasing subsequence of $n + 1$ terms or an increasing subsequence of $n + 1$ terms.

Determine the number a_n of n digit numbers with only odd digits so that the number of times each of 1 and 5 occurs is a multiple of 3.

Use the generating function to find the solution to the following recurrence relations.

(a) Compute a_n where $a_0 = 1$, $a_1 = -2$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$.

(b) Compute the **Catalan number** C_n where $C_1 = 1$, and

$$C_n = \sum_{k=1}^{n-1} C_k C_{n-k} \qquad \text{for} \quad n = 2, 3, \ldots .$$

Find the number of nonnegative integer solutions of

$$x_1 + 2x_2 + 3x_3 + 6x_4 = n$$

with

$$0 \leq x_1 \leq 1, \qquad \text{and} \qquad 1 \leq x_2 \leq 3.$$