Multi-Attribute Decision Making (MADM)

This decision method assumes certainty. In other words, there are no probabilities of future states to determine. And the data and costs are assumed to be known and accurate. The most common type of decision is a preference decision. The decision maker wants to determine which of several options is the best to achieve some set of goals or fulfill a set of criteria or attributes. Common examples are: deciding which car to buy, which house to buy, which apartment to rent, where to go on vacation, which machine to buy for production, which supplier to use, and many more.

The decision process consists of the Decision Maker (DM) identifying the need for some object (or person) or concept that he/she currently does not have. Or it could be to replace some object that has outlived its usefulness, such as replacing a copying machine.

The decision consists of determining a set of criteria that the object must have or meet with some level of satisfaction. For example, when buying a car, the DM might consider its price, color, fuel efficiency, safety rating, warranty, comfort/ride, among other factors. This process is important because it provides and defines the performance and outputs that the user will expect.

The step for this decision is to search for and find the choices (alternatives or options) to be considered. There may be one criteria that is used as a filter, such as price. In the car buying example, the DM may have a price range that fits into his/her budget. They may also have a preference of Make, such as Chevrolet or Ford. But this second preference may actually be a bias and could limit the choices and exclude some viable choices. The search for alternatives usually generates choices in a serial manner. Specific alternatives are identified one at a time. Although it is possible to find several choices at nearly the same time, for example, being shown several different makes and models of cars at one dealership during a single trip.

The DM now has identified the choice options as well as the criteria to be fulfilled. Each alternative will fulfill each criterion at some level of value. The DM must collect this data and put it into a table for easy analysis. Here is an example of a decision table for purchasing a car.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Price | Fuel Effic. (MPG) | Safety Rating | Comfort/Ride | Color |
| Prexel | $22,000 | 32 | 8.5 | 6.7 | Red |
| Criston | $25,000 | 38 | 8.2 | 7.9 | Black |
| Thrush | $27,000 | 35 | 9.6 | 9.2 | Blue |

Note that the names are fictitious. The safety ratings and comfort/ride ratings could easily be obtained from a car buyer magazine. Price and MPG are from the dealerships. We are using only 3 options and 4 criteria for example purposes. The colors are those of cars that are in stock. You could order a car of your preferred color, but you do not want to wait 6 – 8 weeks for delivery.

As you look at this table, you will see that each criterion is measured differently than the others. How do you compare price with MPG, with Safety rating, and with color? To do this, you need a common metric and one that each criterion’s value can be converted easily. This metric is Utility, which is scaled between 0 and 1. Utility of 0 has no or minimal value and utility of 1 is the maximum.

Let’s take a minute to get rigorous in our model and use some shorthand notation.

Let Ci be the ith criterion. We have five criteria and we number them from 1 to 5. So Price is C1 and Color is C5. Criterion Ci, i = 1 to n, and n =5.

Let Aj be the jth alternative. We have three alternative and we number them from 1 to 3. A1 is the Prexel, A2 is the Criston, and A3 is the Thrush. Alternative Aj, j = 1 to m, and m = 3.

In the table we have the values of each alternative j for each criteria i and we term this vij, the value of the ith criteria for the jth alternative. Value vij, i = 1 to n, and j = 1 to m.

But we need to convert the values, vij to utilities, uij, so that they are all measured on the same metric. This will allow us to compare alternatives.

To convert from raw values to utilities can be done in several different ways. The easiest way is to use a linear transformation. Take fuel efficiency. There is 38, 35, and 32. The maximum value is assigned a Utility value of 1 and the minimum value is assigned a Utility value of 0. The intermediate values are assigned a proportionate level using a linear translation. So Utility of 38, or U(38) = 1 and U(32) = 0. Or in general, U(Max value) = 1, and U(Min value) = 0. But what about U(35) = ??

To find U(35), use a translation formula: U(X) = (X – Min value) / (Max value – Min value)

In our example, U(35) = (35 – 32) / (38 – 32) = 3 / 6 = 0.5

This formula for U(X) is for a criterion when More is Better. You can use this to convert the Safety Rating and the Comfort/Ride rating, because More is Better.

But for a criterion where Less is Better, like Price, you need to use this formula:

U(Max value) = 0, and U(Min value) = 1. (remember, Less is Better).

U(X) = (Max value – X) / (Max value – Min Value).

In our example, on the Price criterion: U(27000) = 0, U(22000) = 1,

and U(25000) = (27 – 25) / (27 – 22) = 0.4

But what about criterion that are subjective or do not have any numeric raw values, like color? The utility scores are determined strictly by personal preference. Which of the three colors is most preferred and which is least preferred, and which are in the middle?

In our example, let’s say that Red is most preferred and Black is least preferred, making Blue with a medium level preference. U(Red) = 1, U(Black) = 0, and U(Blue) = ?? Where does Blue fit on a scale of 0 to 1? This is a subjective rating. You can choose any score. In our example the DM prefers Blue to be 0.7, closer to Red than to Black.

Now we have our utilities, uij. Here is the decision table with utility scores.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Price | Fuel Effic. (MPG) | Safety Rating | Comfort/Ride | Color |
| Prexel | 1 | 0 | 0.21 | 0 | 1 |
| Criston | 0.40 | 1 | 0 | 0.48 | 0 |
| Thrush | 0 | 0.50 | 1 | 1 | 0.70 |

There is one more step. Each criterion must be weighted according to its relative importance to the decision or the overall performance or results. These weights are decimals or percent and must total to 1.0. We will let wi denote the numerical weight for each of the i criterion. And we use this formula to insure the correct amount of total weight. Sum(wi) = 1.0.

How does the DM determine these weights? This is subjective as well. The first step is to rank order the criteria with 1 most important and N as the least important (here N = 5). In our example, the DM thinks that Fuel Efficiency and Safety Rating are the most important, but not sure which is first. Then Comfort/Ride. Finally, the DM thinks that maybe price and color are last. The rank ordering is:

[C2 and C3], C4, [C1 and C5].

The DM decides to use the following weights, at least as a starting point. Then he/she can do some sensitivity analysis and adjust them a bit.

C1 = 0.1

C2 = 0.3

C3 = 0.3

C4 = 0.2

C5 = 0.1

Note that these weights total to 1.0.

The final step is to multiply the weights times each utility score for each alternative and sum these to get a total score for each alternative. Using our notation, the formula for Total Score, Tj , for the jth alternative is:

Tj = Sum[(wi)(uij)], i = 1 to n (n = 5)

For example, the total score for the Prexel is calculated as: (0.1)(1) + (0.3)(0) + (0.3)(0.21) + (0.2)(0) + (0.1)(1) = 0.263.

This is the table with all alternatives and their Total Scores using the weights above.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Weight | 10% | 30% | 30% | 20% | 10% | 100% |
|  Criterion | Price | Fuel Effic. (MPG) | Safety Rating | Comfort/Ride | Color | Total Score |
| Prexel | 1 | 0 | 0.21 | 0 | 1 | 0.263 |
| Criston | 0.4 | 1 | 0 | 0.48 | 0 | 0.436 |
| Thrush | 0 | 0.5 | 1 | 1 | 0.7 | 0.720 |

We should not be surprised that that Thrush is the preferred choice. It is scored the best on two of the criteria and 2nd best on two others. And is only scored the worst on one criteria which was given the lowest weight of 0.1

The Prexel scored best on two criteria, but these were the least important in weight and it also scored the worst on two criteria.

The question now to consider, “do these weights reflect the true preference and importance to the DM? If they were adjusted up or down by some degree, would it change the overall total score and hence the decision?”

This process is Sensitivity Analysis.

Watch the video that reviews this resource and discusses Sensitivity Analysis.