Maxwell's equations for a homogeneous, isotropic, uncharged medium of conductivity σ, relative permittivity ε_r and relative permeability μr lead to the expression

$$\mathrm{curl}(\mathrm{curl}\ E) = -\varepsilon r \varepsilon 0 \mu r \mu 0 \frac{d^2 E}{dt^2} - \mu r \mu 0 \sigma \frac{d E}{dt} \qquad (1)$$

where E is the electric field. A linearly polarised plane wave of angular frequency ω is travelling in air in the positive z direction and its electric field is in the x direction. The radiation is incident normally on a conducting medium at z = 0. In the low frequency limit, the equation above reduces to

$$\frac{d^2 Ex}{dz^2} = -\mu r \mu 0 \sigma \frac{d Ex}{dt} \qquad (2) \qquad \text{(all the derivatives are partial)}$$

Show that this equation has solutions of the form $Ex = Et \exp(i(\omega t - \beta z)) \exp(-\alpha z)$

and hence, or otherwise, show that the impedance of the material is $Z = \sqrt{\frac{2\omega\varepsilon 0}{\sigma}} \frac{Z0}{1-i}$

where Z0 is the impedanze of free space.