

6. The equation

$$Ly(x) = \lambda y(x)$$

is a Sturm-Liouville equation in which the operator $L \equiv \frac{d}{dx} \left[p \frac{d}{dx} \right] - q$, where $p(x)$ and $q(x)$ are real functions of x . Any two real solutions $y_m(x)$, $y_n(x)$ with distinct eigenvalues λ_m , λ_n satisfy the boundary condition

$$\left(y_m p \frac{dy_n}{dx} \right)_{x=a} = \left(y_m p \frac{dy_n}{dx} \right)_{x=b}.$$

Show that

$$\int_a^b y_m(x) y_n(x) dx = 0$$

for $n \neq m$.