9. An electron moving with non-relativistic velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ experiences a magnetic field $\mathbf{B}$ given by:

$$\mathbf{B} = -\frac{\mathbf{v} \times \mathbf{E}}{c^2} = -\frac{\mathbf{v} \times (-\nabla \phi(r))}{c^2} ,$$

where $\phi(r)$ is the electric potential. This magnetic field interacts with the magnetic moment μ of the electron given by

$$\mu = -\frac{e}{m_e}\mathbf{S},$$

where $\mathbf{S}$ is the electron spin. Assuming non-relativistic mechanics, show that the Hamiltonian representing this effect (spin-orbit coupling) for a spherically-symmetric electric potential is:

$$\Delta H = -\frac{e}{{m_e}^2 c^2}\,\mathbf{S}\cdot\mathbf{L}\,\frac{1}{r}\frac{\mathrm{d}\phi(r)}{\mathrm{d}r} .$$

Because of Thomas precession, ΔH is divided by a factor of two. Taking only the effect of the spin-orbit coupling into account in the hydrogen atom, calculate the energy splitting of the $n = 2$ excited state and the $n = 3$ excited state assuming that the electron could be in any of the allowed eigenstates of orbital angular momentum. Draw a diagram clearly showing the splittings and label the energy levels using spectroscopic notation.

Comment on the difference between this diagram and that for a real hydrogen atom.

$$\left[\ \left\langle \frac{1}{r^3} \right\rangle = \left(\frac{m_e c \alpha}{\hbar}\right)^3 \frac{2}{n^3 l(l+1)(2l+1)}\ .\ \right]$$