

#4. A toy model for DNA molecules. A DNA molecule is made of two complementary strands, which are hold together by hydrogen bonds. A very simple model a DNA molecule is that of a zipper which has  $N$  links; each link has a state in which it is closed with energy 0 and a state in which it is open with energy  $\varepsilon$ . We require that the zipper only unzip from one side (say from the left) and the link can only open if all the links to the left of it (1, 2, ...,  $n - 1$ ) are already open.

a Find the partition function. (Hint: you might need the following identity

$$\sum_{n=0}^N x^n = \frac{1 - x^{N+1}}{1 - x}. \quad (4)$$

b ) Find the average number of open links  $\langle n \rangle$  and show that for low temperature  $k_B T \ll \varepsilon$ ,  $\langle n \rangle$  is independent of  $N$ .