#3. **Classical vs. Quantum Harmonic Oscillators.** The Hamiltonian of a single harmonic oscillator with angular frequency ω is

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \tag{1}$$

where m is the mass the particle.

a) Using the canonical ensemble, calculate the Helmholtz free energy, the average energy, and the specific heat of a system of N independent classical harmonic oscillators.

b For *quantum* harmonic oscillators, the energy becomes quantized, and the energy spectrum for each oscillator is

$$E_n = n\hbar\omega, \tag{2}$$

up to an irrelevant constant, and n takes on the values of $0, 1, 2, 3, \ldots$ Using the canonical ensemble, calculate the Helmholtz free energy, the average energy, and the specific heat of a system of N independent *quantum* harmonic oscillators. (HINT: you might need the following mathematical identity

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1+x}. \tag{3}$$

c) Show that when temperature is high, the specific heat of a system of quantum oscillators reduces to that of a system of classical oscillators.