Show that the change in internal energy of a simple system between states (V1, T1) and (V2, T2) is given by

$$\Delta U = \int_{T1}^{T2} Cv\, dT + \int_{V2}^{V1} \left[T \left(\frac{dP}{dT} \right)_v - p \right] dV$$

Boyle observed that the pressure of an ideal gas varies inversely with its volume at constant temperature. Give the most general possible equation of state which is consistent with this finding. Explain why the Joule expansion experiment implies that $\left(\frac{dT}{dV}\right)_U = 0$.

Show further that these results taken together imply that the equation of state is pV= RT where R is a constant of proportionality.

A container is thermally isolated and is initially evacuated. The tap linking the container to the outside atmosphere at temperature T0 is now opened and air is allowed to enter slowly until pressure equilibrium is reached. The air which ends up inside the container is assumed to behave as an indeal gas and the thermal capacity of the container can be neglected. Show that the final temperature of the air in the container is γT0, where γ is the ratio of the principal heat capacities of air.

Give a brief statement of the microscopic mechanism of this temperature change.