The function f(x,y) has a turning point at the origin, x = y = 0. Near to the origin the change $\Delta f(x,y)$ in its value can be approximated by

$$\Delta f(x,y) = f(x,y) - f(0,0) = (1/2)(x \;\; y) \begin{pmatrix} \dfrac{d^2 f}{dx^2} & \dfrac{d^2 f}{dxdy} \\ \dfrac{d^2 f}{dydx} & \dfrac{d^2 f}{dy^2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

approximately

$$= \mathbf{x}^{\mathbf{T}}\mathbf{B}\mathbf{x}$$

The eigenvalues of **B** are λ_1 and λ_2. Calculate the product $\lambda_1\lambda_2$ in terms of derivatives of f(x,y) and discuss the nature of the turning point of the function for the cases $\lambda_1\lambda_2 < 0$ and $\lambda_1\lambda_2 > 0$

(all the derivatives are partial)