By considering appropriate series expansions, prove that

$$e^x \cdot e^{x^2/2} \cdot e^{x^3/3} \cdot \ldots = 1 + x + x^2 + \cdots \quad \text{when } |x| < 1.$$

By expanding each individual exponential term on the left-hand side and multiplying out, show that the coefficient of x^{19} has the form

$$1/19! + 1/19 + r/s,$$

where 19 does not divide s. Deduce that

$$18! \equiv -1 \pmod{19}.$$