**3a.**

A solid shaft of diameter $30mm$ rotating at $1000rpm$ delivers a power of $25kW$ determine (assume Shear Modulus for the shaft, $G=80 GPa$)

1. The Shear Stress in the shaft
2. The angular twist of the shaft as a result of the Shear Stress from part (i)
* The important parameters to note here are

 Power, $P=25kW=25,000W$

 Diameter of shaft (needs to be in metres), $D=30mm=0.03m$

 Radius of shaft, $r=\frac{D}{2}=0.015m$

 Rotational speed (needs to be in radians per second),

 $ω=2π×1000rpm=2000π$ rads/s

 Shear Modulus, $G=80 GPa=8×10^{10} Pa$

* (i) First we need to use the Power-Torque equation {1} to determine the Torque in the shaft

$P=ωT$ {1}

 So re-arranged to derive the Torque in the shaft

$$T=\frac{P}{ω}=\frac{25,000}{2000π}=3.978 N.m$$

 We now need to use the equation {2} for the polar second moment of area,

 $ J\_{solid}$ for a solid shaft

$J\_{solid}=\frac{πD^{4}}{32}$ {2}

$$J\_{solid}=\frac{π\left(0.03\right)^{4}}{32}=7.95×10^{-8} m^{4}$$

 One now knows $T,J\_{solid}$ and shaft radius $r$ so we use the Torque-Shear

 Stress, equation {3} to determine the maximum Shear stress, $τ$ in the shaft

$\frac{T}{J\_{solid}}=\frac{τ}{r}$ {3}

$$τ=\frac{Tr}{J\_{solid}}=\frac{3.978×0.015}{7.95×10^{-8}}=7.51×10^{5} N.m^{-2}=751 kN.m^{-2}$$

* (ii) For the angle of twist we need to use the Angle (in radians)-Shear stress equation{4} to determine angular twist in the shaft over length, $L$ , for Shear Modulus, $G=80 GPa$

$θ=\frac{τL}{Gr}$ {4}

$θ=\frac{7.51×10^{5}×2}{8×10^{10}×0.015}=1.25×10^{-3}$ rads

 You can convert to degrees if you want noting the conversion factor ($2π= 360^{0}$

$$θ=\left(\frac{360}{2π}\right)1.25×10^{-3}=0.072^{0} $$

**3b.**

Find the diameter of a solid shaft resulting in a transmission Torque of $50 kNm$ for a maximum allowable Shear Stress in the shaft of $100 MPa$

The important parameters to note here are

Transmission Torque, $T=50kNm=50,000 Nm$

Maximum allowable Shear stress in shaft $τ=100 MPa=10^{8} Pa$

* We need to use the equation {2} for the polar second moment of area,

 $J\_{solid}$ for a solid shaft

$J\_{solid}=\frac{πD^{4}}{32}$ {2}

 And equation {3} relating Shear stress to Torque and radius of shaft

$\frac{T}{J\_{solid}}=\frac{τ}{r}$ {3}

 Noting that radius $r=\frac{D}{2}$ and substituting {2} in {3} we get

$\frac{T}{^{πD^{4}}/\_{32}}=\frac{τ}{^{D}/\_{2}}$ {5}

 Simplifying {5}

$$\frac{32T}{πD^{4}}=\frac{2τ}{D}$$

$\frac{32T}{πD^{3}}=2τ$ {6}

 Rearranging {6} to make $D$ the subject

$D^{3}=\frac{16T}{πτ}$ {7}

 Putting in given values to {7} we get

$$D^{3}=\frac{16×50,000}{π×10^{8}}=2.55×10^{-3}m^{3}$$

$$D=\sqrt[3]{2.55×10^{-3}}=13.7 cm$$

**3c.**

Calculate the power delivered by a rotating hollow shaft with an outer diameter of $150mm$, an inner diameter of $100mm$, rotating at a speed of $3 rps$ for maximum Shear Stress of $50 MPa$

The important parameters to note here is that the shaft is hollow so we need to use equation {8} defining the polar second order of area for a hollow tubular shaft of outer diameter $D$ and inner diameter $d$

$J\_{tube}=\frac{π\left(D^{4}-d^{4}\right)}{32}$ {8}

We are told outer diameter, $D=150mm=0.15m$

Inner diameter, $d=100mm=0.1m$

Using {8} then

$$J\_{tube}=\frac{π\left(0.15^{4}-0.10^{4}\right)}{32}=4×10^{-5}m^{4}$$

We can now use equation {3} to work out the maximum Torque, $T$ on the shaft for maximum Shear stress, $τ=50MPa=5×10^{7}Pa$

$\frac{T}{J\_{tube}}=\frac{τ}{r}$ {3}

 $T=\frac{τJ\_{tube}}{r}$

Putting in values noting that the value radius value, $r$ in the above relates to the radius of the outer diameter which is $r=\frac{D}{2}=\frac{0.15}{2}=0.075m$

Maximum Torque $T=\frac{5×10^{7}×4×10^{-5}}{0.075}=2.67×10^{4}N.m$

We now can use equation {1} to determine the maximum power delivered by this shaft where the given angular velocity of shaft is $ω=2πN$ and where $N=3 rps $

$P=ωT$ {1}

$P=2π×3×2.67×10^{4}=503.3kW$