8. A particle is confined by a 2-dimensional infinite square-well potential of the form:

$$V(x,y) = \begin{cases} 0 & 0 < x < a \text{ and } 0 < y < b \\ \infty & \text{elsewhere} , \end{cases}$$

where $a \leq b$. Solve the time-independent Schrödinger equation for a particle in this potential giving expressions for the normalised eigenfunctions of energy and the energy levels in terms of the quantum numbers n_x and n_y. What happens to the first excited state if $a = b$?

A series of such particles are prepared independently, each in its own potential well as defined above, with $a < b$. The energy of each particle is measured individually. The average of 1000 such energy measurements is very close to the value $E_0 + (E_1 - E_0)/4$, where E_1 is the energy of the first excited state and E_0 is the energy of the ground state of this system. None of the 1000 energy measurements has $E > E_1$. The experimental uncertainty in the energy measurement is negligible. Write down a possible wave function for this potential well that would result in the values obtained from the experiment described. Is this function uniquely specified by the observations?

The measurement is repeated on one additional particle, prepared in the same initial state. What is the probability of finding the particle in the ground state of the system?

What can be said about the probability of finding the particle in the second excited state E_2 given that only 1000 measurements have been made?