4. Give a property that an operator $\hat{\mathrm{A}}$ must have if it represents the physical quantity A of a system described by quantum mechanics. Two such operators $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ represent the quantities A and B of a system. $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ commute. The states $|a, b\rangle$, where $a = 1, 2, 3$ and $b = 6, 7$ are a complete set of simultaneous eigenstates of $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$, and $a\hbar$ and $b\hbar^2$ are the corresponding eigenvalues. Write down the most general state vector which describes the quantities A and B for the system.

The system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{6}}|1, 6\rangle + \frac{2}{\sqrt{6}}|2, 7\rangle + \frac{1}{\sqrt{6}}|3, 7\rangle.$$

What values could result from successive measurements of A then B and what would their probabilities be? Explain whether or not the results would be different if the order of measurement were reversed.