

2. (4 points) A transcendental irrational number

$$S = \sin 0.2 \text{ rad}$$

can be represented by the infinite series

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{(2n+1)}(2n+1)!}$$

Let its partial sum be

$$S_N = \sum_{n=0}^N \frac{(-1)^n}{5^{(2n+1)}(2n+1)!}$$

- Use the Ratio test to show the series converges absolutely.
- Write down explicitly and compute the second (S_2) partial sums.
Hint: in this case, S_2 has three terms (using $n = 0, 1, 2$).

$$S_2 =$$

- Compute the theoretical error bound of the approximation $S \approx S_2$.

$$E_b =$$

- Use your computer or calculator to compute the proxy for exact value of $\sin 0.2$. Observe the theoretical error bound holds for S_2 . What does this fact mean?