$\mathbf{R}$ is a 3×3 matrix. What property is required of $\mathbf{R}$ for it to be (a) an orthogonal matrix, and (b) a rotation matrix? Write down the matrix that effects a rotation by angle θ about the y axis.

A 3×3 matrix $\mathbf{M}$ maps vectors $\mathbf{a} \to \mathbf{b}$ by $\mathbf{b} = \mathbf{M} \cdot \mathbf{a}$. Find an expression for the matrix $\mathbf{M}'$ that maps the rotated vectors $\mathbf{a}' = \mathbf{R} \cdot \mathbf{a}$ and $\mathbf{b}' = \mathbf{R} \cdot \mathbf{b}$ into one another.

Prove that $\operatorname{trace}(\mathbf{M}') = \operatorname{trace}(\mathbf{M})$.

Find the rotation axis of the rotation matrix

$$\begin{pmatrix} 1/2 & -1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}.$$