††**Particle in a magnetic field**

The Hamiltonian for a particle of charge q, mass m_0, in a constant magnetic field $\boldsymbol{B} = (0, 0, B)$ is (approximately)

$$H = -\frac{\hbar^2 \boldsymbol{\nabla}^2}{2m_0} - \frac{qB}{2m_0} L_z.$$

(a) The eigenvalues of which of the following are good quantum numbers: $\boldsymbol{L}^2$, L_x, L_y, L_z?

(b) Show (using the Ehrenfest Theorem result from MT) that the expectation values of L_x, L_y and L_z in a general state $|\psi\rangle$ satisfy the equations

$$\begin{aligned} \frac{d}{dt}\langle\psi|L_x|\psi\rangle &= \frac{qB}{2m_0}\langle\psi|L_y|\psi\rangle \\ \frac{d}{dt}\langle\psi|L_y|\psi\rangle &= -\frac{qB}{2m_0}\langle\psi|L_x|\psi\rangle \\ \frac{d}{dt}\langle\psi|L_z|\psi\rangle &= 0. \end{aligned} \tag{12}$$

(c) The magnetic moment operator $\boldsymbol{\mu}$ is defined by $\boldsymbol{\mu} = \frac{q}{2m_0}\boldsymbol{L}$. Deduce that

$$\frac{d}{dt}\langle\psi|\boldsymbol{\mu}|\psi\rangle = \frac{q}{2m_0}\langle\psi|\boldsymbol{\mu}|\psi\rangle \times \boldsymbol{B}. \tag{14}$$

(d) You've seen the equation of motion (14) before in the Prelims vector work. Show that (i) the length of the vector $\langle\psi|\boldsymbol{\mu}|\psi\rangle$ is constant in time, and (ii) $\langle\psi|\boldsymbol{\mu}|\psi\rangle.\boldsymbol{B}$ is constant in time. Hence show that $\langle\psi|\boldsymbol{\mu}|\psi\rangle$ precesses around the direction of $\boldsymbol{B}$ with angular frequency $\omega = \frac{qB}{2m_0}$. Calculate ω for the electron in a field of 1 tesla.