Question 1

If [K:F] is finite and u is algebraic over K ,prove that [F(u):F] divides [K(u):F]

Hint:[F(u):F] and [K(u):F(u)] are finite by Theorems 10.4,10.7 and 10.9

Apply Theorem 10.4 to $F⊆F\left(u\right)⊆K(u)$

Theorem 10.4

Let F,K and L be fields with F ⊆ K ⊆ L .If [K:F] and [L:K] are finite ,then L is a finite dimensional extension of F and [L:F] = [L:K] [K:F]

Theorem 10.7

Let K be an extension field of F and u $\in K $ an algebraic element over F with minimal polynomial p(x) of degree n.Then

1. F(u) $≅$ F[x]/(p(x))
2. {$1\_{F}$,u ,$u^{2}$ , ……………..,$u^{n-1}$} is a basis of the vector space F(u) over F
3. [F(u):F] = n

Theorem 10.7 shows that when u is algebraic over F,then F(u) does not depend on K but is completely determined by F[x] and the minimal polynomial p(x).Consequently ,we sometimes say that F(u) is the **field obtained by adjoining u to F**

Theorem 10.9

If K is a finite-dimensional extension field of F,then K is an algebraic extension of F.

Question 2

Assume that u,v $\in $ K are algebraic over F,with minimal polynomial p(x) and q(x),respectively.

1. If deg p(x) = m and deg q(x) = n and (m,n) = 1, prove that [F(u,v):F] = mn.
2. Show by example that the conclusion of part (a) may be false if *m* and *n* are not relatively prime.
3. What is [**Q**($\sqrt{2} ,\sqrt[3]{2}$):**Q**]?