1.

The ground-state energy of a quantum particle of mass m in a pillbox (right-circular cylinder) is given by

$$E = \frac{\hbar^2}{2m}\left(\frac{(2.4048)^2}{R^2} + \frac{\pi^2}{H^2}\right),$$

in which R is the radius and H is the height of the pillbox. Find the ratio of R to H that will minimize the energy for a fixed volume.

2.

A particle, mass m, is on a frictionless horizontal surface. It is constrained to move so that $\theta = \omega t$ (rotating radial arm, no friction). With the initial conditions

$$t = 0, \qquad r = r_0, \qquad \dot{r} = 0,$$

(a) find the radial positions as a function of time.

ANS. $r(t) = r_0 \cosh \omega t$.

(b) find the force exerted on the particle by the constraint.

ANS. $F^{(c)} = 2m\dot{r}\omega = 2mr_0\omega^2 \sinh \omega t$.

3.

A point mass m is moving over a flat, horizontal, frictionless plane. The mass is constrained by a string to move radially inward at a constant rate. Using plane polar coordinates (ρ, φ), $\rho = \rho_0 - kt$,

(a) Set up the Lagrangian.
(b) Obtain the constrained Lagrange equations.
(c) Solve the φ-dependent Lagrange equation to obtain $\omega(t)$, the angular velocity. What is the physical significance of the constant of integration that you get from your "free" integration?
(d) Using the $\omega(t)$ from part (b), solve the ρ-dependent (constrained) Lagrange equation to obtain $\lambda(t)$. In other words, explain what is happening to the **force** of constraint as $\rho \to 0$.