Derivation of Radial Equation

Now, we have seen that the Cartesian components of the momentum, $\mathbf{p}$, can be represented as (see Sect. 7.2)

$$p_i = -\mathrm{i}\,\hbar\,\frac{\partial}{\partial x_i} \tag{624}$$

for $i = 1, 2, 3$, where $x_1 \equiv x$, $x_2 \equiv y$, $x_3 \equiv z$, and $\mathbf{r} \equiv (x_1, x_2, x_3)$. Likewise, it is easily demonstrated, from the above expressions, and the basic definitions of the spherical polar coordinates [see Eqs. (545)-(550)], that the radial component of the momentum can be represented as

$$p_r \equiv \frac{\mathbf{p}\cdot\mathbf{r}}{r} = -\mathrm{i}\,\hbar\,\frac{\partial}{\partial r}. \tag{625}$$

Recall that the angular momentum vector, $\mathbf{L}$, is defined [see Eq. (526)]

$$\mathbf{L} = \mathbf{r}\times\mathbf{p}. \tag{626}$$

This expression can also be written in the following form:

$$L_i = \epsilon_{ijk}\,x_j\,p_k. \tag{627}$$

Here, the ϵ_{ijk} (where i, j, k all run from 1 to 3) are elements of the so-called *totally anti-symmetric tensor*. The values of the various elements of this tensor are determined via a simple rule:

$$\epsilon_{ijk} = \left\{\begin{array}{cl} 0 & \text{if } i,j,k \text{ not all different} \\ 1 & \text{if } i,j,k \text{ are cyclic permutation of } 1,2,3 \\ -1 & \text{if } i,j,k \text{ are anti-cyclic permutation of } 1,2,3 \end{array}\right. . \tag{628}$$

Thus, $\epsilon_{123} = \epsilon_{231} = 1$, $\epsilon_{321} = \epsilon_{132} = -1$, and $\epsilon_{112} = \epsilon_{131} = 0$, *etc.* Equation (627) also makes use of the *Einstein summation convention*, according to which repeated indices are summed (from 1 to 3). For instance, $a_i\,b_i \equiv a_1\,b_1 + a_2\,b_2 + a_3\,b_3$. Making use of this convention, as well as Eq. (628), it is easily seen that Eqs. (626) and (627) are indeed equivalent.

Can you show all the steps that proves (626) and (627) are equivalent? Thank you