1. **Positively Curved Spaces:** Imagine that the Universe has 2 space dimensions and 1 time dimension, and is described by the metric:

$$ds^2 = -c^2dt^2 + dr^2 + R^2 \sin^2(r/R)\, d\theta^2 \tag{1}$$

with $-\infty < t < \infty$, $0 < r < \pi R$, and $0 < \theta < 2\pi$.

   (a) A photon is released from $\{t = 0,\ r = r_0,\ \theta = 0\}$ in the θ-direction. It does not move in the r-direction. What is the trajectory of the photon $\theta(t)$?

   (b) What is the light travel time from the south pole at $r = \pi R$ to the north pole at $r = 0$?

2. **Negatively Curved Spaces:** Imagine that the Universe has 2 space dimensions and 1 time dimension, and is described by the metric:

$$ds^2 = -c^2dt^2 + dr^2 + R^2 \sinh^2(r/R)\, d\theta^2 \tag{2}$$

with $-\infty < t < \infty$, $0 < r < \infty$, and $0 < \theta < 2\pi$.

   (a) What is the circumference of a circle of radius $r = r_0$?

   (b) The spatial area dA in an annulus of width dr is:

$$dA = C dr \tag{3}$$

   where C is the circumference you found above. Integrate this expression to find the area of a disc of radius r_0 centred on $r = 0$.

   (c) Find the circumference of a circle of radius r_0 centred on $r = 0$ in a flat and positively curved Universe. Use the relation above to find the area of a disc of radius r_0 and compare with your result for a negatively curved Universe.

   (d) **Bonus:** Imagine that these 2D Universes are filled with galaxies at a constant number density $n_{\rm gal} \equiv$ Number/Unit Area. Assume that the galaxies in your Universe were all formed at time $t = 0$ and that we are living at time t_0. What is the total number of galaxies that you can observe in a flat, positively curved, and negatively curved Universe? This number is different, suggesting that we can determine the curvature by counting the number of galaxies in our observable Universe using the locally observed density of galaxies. Assuming that galaxies are randomly distributed, find the $1-\sigma$ detection limit on R based on counting the number of galaxies, as a function of the number of galaxies observed in a flat Universe. Hint: As for any number count, use the Poisson distribution. **Note: this is not the**