Stochastic Operations Research

Read: Sections 12.3 - 12.5, 13.1, 13.4-13.6 in “Winston, Operations Research: Application and Algorithms, 4th edition, 2004, Thompson, ISBN 0-534-38058-1.”

Question 1

Diamond and Forrester (1979) applied Bayes’ rule to the diagnosis of coronary artery disease. A procedure called cardiac fluoroscopy is used to determine whether there is calcification of coronary arteries and thereby to diagnose coronary artery disease. From the test it can be determined if 0, 1, 2, or 3 coronary arteries are calcified. Let T0, T1, T2, T3, denote these events. Let D+ or D- denote the event that disease is present or absent, respectively. Diamond and Forrester presented the following table based on medical studies.

iP(Ti|D+)P(Ti|D-)

0 0.42 0.96

1 0.24 0.02

2 0.20 0.02

3 0.15 0.00

Medical statistics also supply the likelihood that the disease is present based upon different ages and situations. For a male patient between 30 and 39, presenting with nonanginal chest pain the probability that the disease is present is approximately 5%. In contrast, for a male between ages 50 and 59 who suffers typical angina the probability that the disease is present is 92%. Based on this information please answer the following questions.

a. For a male patient between ages 30 and 39 with nonanginal chest pain, a test is run and no arteries are calcified. What is the probability that the disease is present?

b. For that same younger patient, what is the probability if the test had shown instead 1 calcified artery?

c. Now for an older male patient between ages 50 and 59 with typical angina what is the probability that the disease is present when no arteries are calcified?

d. For that same older patient what is probability if instead 1 artery was found calcified? e. Compare the answers for parts a. to d and how D+ affects the final probabilities.

Question 2

After the apocalypse, three species, zombies, vampires and aliens could kill a human with probabilities 0.02, 0.03, and 0.05 if they attack. The probability of a zombie attack is 0.5, a vampire attack is 0.3 and an alien attack is 0.2. If a person is killed, what is the probability he/she is attacked by a zombie?

Question 3

Football teams have multiple players to fill the same position in case one player gets injured. A team needs to decide how many players “deep” to have at a position, how many backups they might need. Consider a team that is three deep at the wide receiver position. There is a 15% chance that only the starting player will get injured. There is another 10% chance that both the starting player and the secondary player will get injured. There is a 5% chance that all three players will get injured. On average, how many of their wide receivers do they actually use in the season?

Question 4

A woman is trapped in a chamber with two doors. The first door leads to freedom in 2 hours. The second door leads to a second chamber in 1 hour. In the second chamber there are three doors. The first door leads back to the first chamber in 2 hours. The second door leads into the second chamber in 3 hours. The third door leads to freedom in 1 hour. How long will it be before the woman reaches freedom? How many doors will she have to try?

Question 5

Suppose that Papa John’s and Domino’s stop advertising but must determine the price they will charge for each pizza sold. Papa John’s believes that Domino’s price is a random variable “D” having the following mass function: P(D = $6) = 0.25, P(D = $8) = 0.5, P(D = $10) = 0.25. If Papa John’s charges a price “p1” and Domino’s charges a price “p2”, Papa John’s will sell (100+25\*(p2 – p1)) pizzas. It costs Papa John’s $4 to make a pizza. Papa John’s is considering charging $5, $6, $7, $8, or $9 for a pizza. Use the decision criterions (maximin, maximax, minimax regret and expected payoff) to determine the price Papa John’s should charge.

Question 6

Jose Morales manages a large outdoor fruit stand in one of the less affluent neighborhoods of San Jose, California. To replenish his supply, Jose buys boxes of fruit early each morning from a grower south of San Jose. About 90 percent of the boxes of fruit turn out to be of satisfactory quality, but the other 10 percent are unsatisfactory. A satisfactory box contains 80 percent excellent fruit and will earn $200 profit for Jose. An unsatisfactory box contains 30 percent excellent fruit and will produce a loss of $1,000. Before Jose decides to accept a box, he is given the opportunity to sample one piece of fruit to test whether it is excellent. Based on that sample, he then has the option of rejecting the box without paying for it. Jose wonders (1) whether he should continue buying from this grower, (2) if so, whether it is worthwhile sampling just one piece of fruit from a box, and (3) if so, whether he should be accepting or rejecting the box based on the outcome of this sampling. Determine Jose’s optimal strategy. Also find EVPI and EVSI.