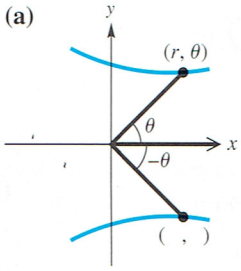
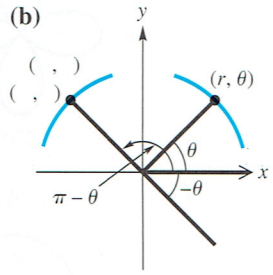
Concept Check *The polar graphs in this section exhibit symmetry.* ***(See Appendix D.)*** *Visualize an xy-plane superimposed on the polar coordinate system, with the pole at the origin and the polar axis on the positive x-axis. Then a polar graph may be symmetric with respect to the x-axis (the polar axis), the y-axis* $\left(\text{the line } \theta = \frac{\pi}{2}\right)$*, or the origin (the pole). Use this information to work Exercises 71 and 72.*

71. Complete the missing ordered pairs in the graphs below.

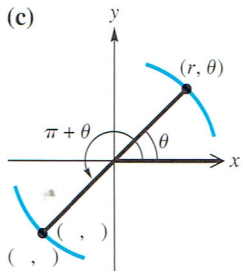
(a)



(b)



(c)



72. Based on your results in **Exercise 71,** fill in the blanks with the correct responses.

(a) The graph of $r = f(\theta)$ is symmetric with respect to the polar axis if substitution of _____ for θ leads to an equivalent equation.

(b) The graph of $r = f(\theta)$ is symmetric with respect to the vertical line $\theta = \frac{\pi}{2}$ if substitution of _____ for θ leads to an equivalent equation.

(c) Alternatively, the graph of $r = f(\theta)$ is symmetric with respect to the vertical line $\theta = \frac{\pi}{2}$ if substitution of _____ for r and _____ for θ leads to an equivalent equation.

(d) The graph of $r = f(\theta)$ is symmetric with respect to the pole if substitution of _____ for r leads to an equivalent equation.

(e) Alternatively, the graph of $r = f(\theta)$ is symmetric with respect to the pole if substitution of _____ for θ leads to an equivalent equation.

(f) In general, the completed statements in parts (a)–(e) mean that the graphs of polar equations of the form $r = a \pm b\cos\theta$ (where a may be 0) are symmetric with respect to _____.

(g) In general, the completed statements in parts (a)–(e) mean that the graphs of polar equations of the form $r = a \pm b\sin\theta$ (where a may be 0) are symmetric with respect to _____.

The graph of $r = a\theta$ in polar coordinates is an example of the spiral of Archimedes. With your calculator set to radian mode, use the given value of a and interval of θ to graph the spiral in the window specified.

73. $a = 1, 0 \le \theta \le 4\pi$, $[-15, 15]$ by $[-15, 15]$

74. $a = 2, -4\pi \le \theta \le 4\pi$, $[-30, 30]$ by $[-30, 30]$

75. $a = 1.5, -4\pi \le \theta \le 4\pi$, $[-20, 20]$ by $[-20, 20]$

76. $a = -1, 0 \le \theta \le 12\pi$, $[-40, 40]$ by $[-40, 40]$

Find the polar coordinates of the points of intersection of the given curves for the specified interval of θ.

77. $r = 4\sin\theta, r = 1 + 2\sin\theta;\ 0 \le \theta < 2\pi$

78. $r = 3, r = 2 + 2\cos\theta;\ 0° \le \theta < 360°$

79. $r = 2 + \sin\theta, r = 2 + \cos\theta;\ 0 \le \theta < 2\pi$

80. $r = \sin 2\theta, r = \sqrt{2}\cos\theta;\ 0 \le \theta < \pi$

(Modeling) Solve each problem.

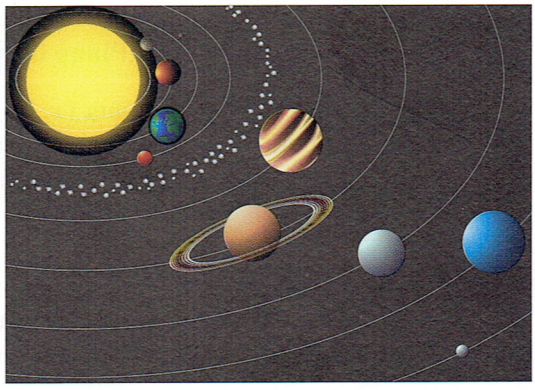
81. ***Orbits of Satellites*** The polar equation

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

can be used to graph the orbits of the satellites of our sun, where a is the average distance in astronomical units from the sun and e is a constant called the **eccentricity**. The sun will be located at the pole. The table lists the values of a and e.

Satellite	a	e
Mercury	0.39	0.206
Venus	0.78	0.007
Earth	1.00	0.017
Mars	1.52	0.093
Jupiter	5.20	0.048
Saturn	9.54	0.056
Uranus	19.20	0.047
Neptune	30.10	0.009
Pluto	39.40	0.249

Source: Karttunen, H., P. Kröger, H. Oja, M. Putannen, and K. Donners (Editors), *Fundamental Astronomy, 4th edition,* Springer-Verlag. Zeilik, M., S. Gregory, and E. Smith, *Introductory Astronomy and Astrophysics,* Saunders College Publishers.



(a) Graph the orbits of the four closest satellites on the same polar grid. Choose a viewing window that results in a graph with nearly circular orbits.

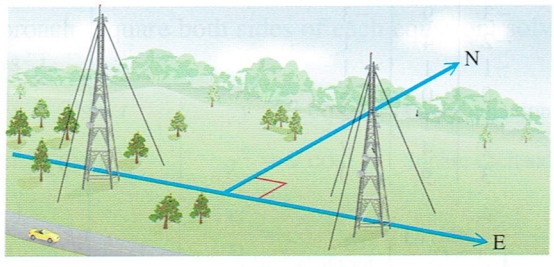
(b) Plot the orbits of Earth, Jupiter, Uranus, and Pluto on the same polar grid. How does Earth's distance from the sun compare to the others' distances from the sun?

(c) Use graphing to determine whether or not Pluto is always farthest from the sun.

82. ***Radio Towers and Broadcasting Patterns*** Many times radio stations do not broadcast in all directions with the same intensity. To avoid interference with an existing station to the north, a new station may be licensed to broadcast only east and west. To create an east-west signal, two radio towers are sometimes used, as illustrated in the figure. Locations where the radio signal is received correspond to the interior of the curve

$$r^2 = 40{,}000 \cos 2\theta,$$

where the polar axis (or positive x-axis) points east.



(a) Graph $r^2 = 40{,}000 \cos 2\theta$ for $0° \le \theta \le 360°$, where distances are in miles. Assuming the radio towers are located near the pole, use the graph to describe the regions where the signal can be received and where the signal cannot be received.

(b) Suppose a radio signal pattern is given by

$$r^2 = 22{,}500 \sin 2\theta.$$

Graph this pattern and interpret the results.