http://ramanujan.math.trinity.edu/wtrench/texts/TRENCH_REAL_ANALYSIS.PDF

The lemma is from page 327 in the link. I am not sure about the dimensions . I believe that **X** is a vector.

Theorem 5.3.7 implies the following lemma, which is analogous to Lemma 2.3.2. We leave the proof to you (Exercise 5.3.13).

Lemma 5.3.8 *If f is differentiable at $\mathbf{X}_0$, then*

$$f(\mathbf{X}) - f(\mathbf{X}_0) = (d_{\mathbf{X}_0} f)(\mathbf{X} - \mathbf{X}_0) + E(\mathbf{X})|\mathbf{X} - \mathbf{X}_0|,$$

where E is defined in a neighborhood of $\mathbf{X}_0$ and

$$\lim_{\mathbf{X} \to \mathbf{X}_0} E(\mathbf{X}) = E(\mathbf{X}_0) = 0.$$

I need a proof for this lemma 5.3.8. And in addition to the proof I need you to define

1: What is **E**(x)

2: what does the notation $\left(d_{x_0} f\right)(\boldsymbol{X} - \boldsymbol{X_0})$ mean. I have never seen the notation $\left(d_{x_0} f\right)$?

3: Please define the dimensions of f and **X** in the proof.