

$$E\left(e^{\left(a\_{1}X\_{1}+a\_{2}X\_{2}+…a\_{n}X\_{n}\right)t}\right)$$

$$=E\left(e^{a\_{1}X\_{1}t}\right)E\left(e^{a\_{2}X\_{2}t}\right)…E\left(e^{a\_{n}X\_{n}t}\right)$$

$$e^{a\_{1}X\_{1}t}=\left(e^{X\_{1}t}\right)^{a\_{1}} moment generating function $$

$$=E\left(\left(e^{X\_{1}t}\right)^{a\_{1}}\right)E\left(\left(e^{X\_{2}t}\right)^{a\_{2}}\right)…E\left(\left(e^{X\_{n}t}\right)^{a\_{n}}\right)$$

$$=\left(E\left(e^{X\_{1}t}\right)\right)^{a\_{1}}\left(E\left(e^{X\_{2}t}\right)\right)^{a\_{2}}…\left(E\left(e^{X\_{n}t}\right)\right)^{a\_{n}}$$

$$=\left(e^{μt+\frac{σ^{2}t^{2}}{2}}\right)^{a\_{1}}\left(e^{μt+\frac{σ^{2}t^{2}}{2}}\right)^{a\_{2}}…\left(e^{μt+\frac{σ^{2}t^{2}}{2}}\right)^{a\_{n}}$$

$$=\left(e^{μt+\frac{σ^{2}t^{2}}{2}}\right)^{a\_{1}+a\_{2}+…+a\_{n}}=\left(e^{μ\left(a\_{1}+a\_{2}+…+a\_{n}\right)t+\frac{\left(a\_{1}+a\_{2}+…+a\_{n}\right)σ^{2}t^{2}}{2}}\right)^{a\_{1}+a\_{2}+…+a\_{n}}$$

This is wrong since it should have variance



And hence

$$=e^{μ\left(a\_{1}+a\_{2}+…+a\_{n}\right)t+\frac{\left(a\_{1}^{2}+a\_{2}^{2}+…+a\_{n}^{2}\right)σ^{2}t^{2}}{2}}$$

The only thing that I could see that is wrong is that

$$E\left(\left(e^{X\_{1}t}\right)^{a\_{1}}\right)\ne \left(E\left(e^{X\_{1}t}\right)\right)^{a\_{1}}$$

I would like to be pointed out what is wrong with my attempt and be given a new derivation that is justified as correct through all the steps.