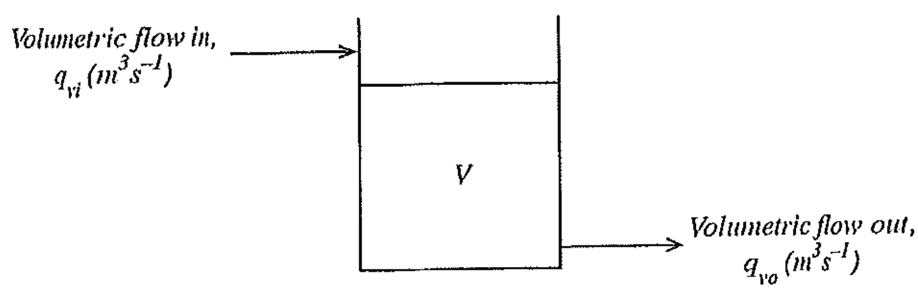
An overall mass balance is needed whenever you are interested in the 'hold-up' of a system. 'Hold-up' (accumulation/depletion) is typically measured in practice by using levels within vessels for liquid systems or pressures for gas/vapour systems.

A mass balance may be written over each system or subsystem that you can define within your process.

Constitutive equations may be needed to define system properties, such as density in terms of composition, temperature, pressure, etc.

As an example, let's consider a liquid storage tank as shown in FIGURE 2. We are going to do an overall mass balance over the tank.



We cannot do a volume balance as the volume of a given mass of the material (density) may change. To do a mass balance we need to convert volume to mass.

$$\text{mass } (m) = \text{density } (\rho) \times \text{volume } (V)$$

and thus, $$\text{mass flow rate } (q_m) = \text{density } (\rho) \times \text{volume flow rate } (q_v)$$

If we let the subscripts $_i$ = input and $_o$ = output

Using the standard mass balance equation, over a given time period:

mass flow in = mass flow out + accumulation (the change in mass within the tank)

Note: accumulation can have a negative value (i.e. mass in tank can reduce).

For unit time,

$$\rho_i q_{vi} = \rho_o q_{vo} + \text{accumulation}$$

So the change in mass within the tank, with respect to time

$$= \frac{dm}{dt} = \frac{d\rho V}{dt} = \rho_i q_{vi} - \rho_o q_{vo}$$

where ρ is the average liquid density within the tank.

If the cross-sectional area of the tank is constant, the volume of liquid in the tank (V) can be determined as:

$$V = \text{cross-sectional area } (A) \text{ of the tank} \times \text{ the height of liquid } (h) \text{ in the tank}$$

We can then determine just one process variable to represent this change in mass, i.e. the change in level of liquid in the tank.

Thus,
$$\frac{d\rho V}{dt} = \frac{A d\rho h}{dt} = \rho_i q_{vi} - \rho_o q_{vo}$$

If the density of the liquid is constant i.e. $\rho_i = \rho_o = \rho$, then we can cancel by ρ and the final part of this equation can be simplified to:

$$\frac{A\mathrm{d}h}{\mathrm{d}t} = q_{vi} - q_{vo}$$

or $$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{(q_{vi} - q_{vo})}{A}$$

To obtain this equation, we have made the following assumptions, which are commonly made when compiling and simplifying mass balances.

Constant density of liquid (with respect to time) – true only if there is no change in temperature and no chemical reaction occurs.

Constant flow rates (with respect to time), i.e. the flow rates in and out do not fluctuate.

When dealing with mixtures of materials, there is perfect mixing so materials do not settle out due to density differences and thus we can use an average density to determine mass.