1. Determine whether the sequence whose nth term is $\ell n n! - (n + \frac{1}{2})\, \ell n n + n$ converges or diverges.

2. (a) Given $f : R \to R$, prove that differentiability implies continuity but not vice versa.

   (b) Consider an interval $I \subset R$ and suppose that $f : I \to R$ is continuous. Prove that if (i) x^* is a local maximum of f and (ii) x^* is the only extreme point of f on I, then x^* is a global maximum of f on I.

3. Determine whether

   (a) reciprocals of positive concave functions are convex,

   (b) reciprocals of positive convex functions are concave,

   (c) there exists a convex function $f : R_{++} \to R$ such that $f(x) \leq \ell n x$ for all $x \in R_{++}$.

4. Suppose that $f(x)$ has a continuous first derivative for all $x \in R$.

   (a) Prove that $f(x)$ is concave if and only if $f(x^*) + (x - x^*)f'(x^*) \geq f(x)$ for all x and $x^* \in R$.

   (b) Given that $f(x)$ is concave, prove that x^* is a global maximum of $f(x)$ if and only if $f'(x^*) = 0$.

   (c) Given that $f(x)$ is concave, prove that its set of global maxima is either empty, a singleton, or an infinite convex set.