a) The gravitational potential energy of a mass due to a mass density satisfies = 4 , where *G* is the gravitational constant. If the earth is considered to be a uniform sphere of mass *M*, radius *R*, show that the gravitational potential energy of a mass *m* *inside* the earth a distance from the center is

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where = = 9.81

b) A tunnel is to be constructed through the earth, along which a frictionless train will run between cities at A and B. The track is described by the curve (polar coordinates).



Use energy conservation to derive an expression for the speed of a train which starts at rest at as it passes through segment of the curve at , and hence show that the journey time is



where

c) The tunnel is to be constructed so as to minimize the journey time. Taking care to note the nature of the integrand, write down an Euler-Lagrange equation for the extremal curve [not asked to solve this].

Let be the minimum distance of the tunnel from the center of the earth, where = 0. Use this to obtain an expression for in terms of and

d) Noting = /), reexpress as an integral



and evaluate.

e) If = 6,400 km determine the journey time in minutes through the center of the earth.