6. A polymer, like DNA, is a molecule composed of a long chain of identical molecular units, called monomers. Therefore, it may be appropriate to model a long polymer, made of N rodlike units, each of length ℓ, attached end-to-end. Assume that the connections between the monomers are completely flexible so that the rods can make any angle with respect to each other. On end of polymer is held fixed while a constant force F, in the x-direction, is applied to the other end. (The is precisely the kind of setup that people use in single molecules experiments, like pulling on DNA to extract structural information about DNA.) Up to a constant, which can be ignored, the potential energy of any configuration of the polymer can be written as

$$U(\theta_1, \phi_1, ..., \theta_N, \phi_N) = -\sum_{i=1}^{N} F\ell \cos\theta_i \tag{7}$$

where θ_i and ϕ_i are polar angles defining the orientation, in three dimensions, of the ith monomer with respect to an axis parallel to the x-axis.

a) Evaluate the configuration partition function Z:

$$Z = \int d\theta_1 \sin\theta_1 d\phi_1 ... \int d\theta_N \sin\theta_N d\phi_N \, e^{-\beta U} \tag{8}$$

b) Calculate the average extension $\langle R \rangle = \ell \sum_{i=1}^{N} \langle \cos\theta_i \rangle$ as a function of F and T, and obtain the high temperature and low temperature approximation of $\langle R \rangle$.