## 6. Charged harmonic oscillator in a variable electric field

A one-dimensional harmonic oscillator is composed of a particle of mass $m$, charge $q$ and potential energy $V(X) = \frac{1}{2} m\omega^2 X^2$. We assume in this exercise that the particle is placed in an electric field $\mathscr{E}(t)$ parallel to $Ox$ and time-dependent, so that to $V(x)$ must be added the potential energy:

$$W(t) = -q\mathscr{E}(t)X$$

*a*. Write the Hamiltonian $H(t)$ of the particle in terms of the operators $a$ and $a^\dagger$. Calculate the commutators of $a$ and $a^\dagger$ with $H(t)$.

*b*. Let $\alpha(t)$ be the number defined by:

$$\alpha(t) = \langle \psi(t) | a | \psi(t) \rangle$$

where $| \psi(t) \rangle$ is the normalized state vector of the particle under study. Show from the results of the preceding question that $\alpha(t)$ satisfies the differential equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} \alpha(t) = -i\omega\, \alpha(t) + i\lambda(t)$$

where $\lambda(t)$ is defined by:

$$\lambda(t) = \frac{q}{\sqrt{2m\hbar\omega}} \mathscr{E}(t)$$

Integrate this differential equation. At time $t$, what are the mean values of the position and momentum of the particle?

*c*. The ket $| \varphi(t) \rangle$ is defined by:

$$| \varphi(t) \rangle = [a - \alpha(t)] \, | \psi(t) \rangle$$

where $\alpha(t)$ has the value calculated in *b*. Using the results of questions *a* and *b*, show that the evolution of $| \varphi(t) \rangle$ is given by:

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} | \varphi(t) \rangle = [H(t) + \hbar\omega] \, | \varphi(t) \rangle$$

How does the norm of $| \varphi(t) \rangle$ vary with time?