Problem 2:

$$2\pm i$$

Gives a second order polynomial:

$$\left(x-2-i\right)\left(x-2+i\right)=x^{2}-2x+xi-2x+4-2i-ix+2i-i^{2}$$

$$=x^{2}-2x-2x+4+1=x^{2}-4x+5$$

$$\frac{x^{3}+3x^{2}+x}{\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}$$

$$\frac{x^{3}+3x^{2}+x}{\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}=\frac{A}{\left(x+2\right)}+\frac{Bx^{2}+Cx+D}{\left(x^{2}-4x+5\right)^{2}}$$

We see that if we multiply with the denumerator we get

$$x^{3}+3x^{2}+x=\frac{A\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}{\left(x+2\right)}+\frac{\left(Bx^{2}+Cx+D\right)\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}{\left(x^{2}-4x+5\right)^{2}}$$

$$x^{3}+3x^{2}+x=A\left(x^{2}-4x+5\right)^{2}+\left(Bx^{2}+Cx+D\right)\left(x+2\right)$$

$$x^{3}+3x^{2}+x=\left(Ax^{4}-4Ax^{3}+5Ax^{2}-4Ax^{3}+16Ax^{2}-20Ax+5Ax^{2}-20Ax+25A\right)+\left(Bx^{3}+Cx^{2}+Dx+2Bx^{2}+2Cx+2D\right)$$

$$x^{3}+3x^{2}+x=Ax^{4}-4Ax^{3}+5Ax^{2}-4Ax^{3}+16Ax^{2}-20Ax+5Ax^{2}-20Ax+25A+Bx^{3}+Cx^{2}+Dx+2Bx^{2}+2Cx+2D$$

$$x^{3}+3x^{2}+x=Ax^{4}-4Ax^{3}+Bx^{3}-4Ax^{3}+5Ax^{2}+16Ax^{2}+5Ax^{2}+Cx^{2}+2Bx^{2}-20Ax-20Ax+Dx+2Cx+25A+2D$$

$$x^{3}+3x^{2}+x=Ax^{4}+\left(-4A+B-4A\right)x^{3}+\left(5A+16A+5A+C+2B\right)x^{2}+\left(-20A-20A+D+2C\right)x+25A+2D$$

We have 5 equations and 4 unknowns

$$A=0 -4A+B-4A=1 5A+16A+5A+C+2B=3 -20A-20A+D+2C=1 25A+2D=0 $$

$$A=0 B-8A=1 26A+C+2B=3 -40A+D+2C=1 25A+2D=0 $$

$$B=1 C+2=3 C=1 D+2=1 D=-1 $$

This is an inconsistent system of equations

$$\frac{x^{3}+3x^{2}+x}{\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}=\frac{A}{\left(x+2\right)}+\frac{Bx^{2}+Cx+D}{\left(x^{2}-4x+5\right)^{2}}$$

$$we want to write Bx^{2}+Cx+D as \left(x^{2}-4x+5\right)$$

$$B\left(x^{2}-4x+5\right)=Bx^{2}-4Bx+5B$$

$$is Bx^{2}+Cx+D=B\left(x^{2}-4x+5\right) ? $$

$$Bx^{2}+Cx+D=Bx^{2}-4Bx+5B$$

$$Cx+D=-4Bx+5B$$

We get a difference:

$$\left(C+4B\right)x and D-5B$$

so

$$Bx^{2}+Cx+D=B\left(x^{2}-4x+5\right)+\left(C+4B\right)x+\left(D-5B\right)$$

$$=Bx^{2}-4Bx+5B+Cx+4Bx+D-5B=Bx^{2}+Cx+D$$

$$\frac{x^{3}+3x^{2}+x}{\left(x+2\right)\left(x^{2}-4x+5\right)^{2}}=\frac{A}{\left(x+2\right)}+\frac{B\left(x^{2}-4x+5\right)+\left(C+4B\right)x+\left(D-5B\right)}{\left(x^{2}-4x+5\right)^{2}}$$

$$=\frac{A}{\left(x+2\right)}+\frac{B}{\left(x^{2}-4x+5\right)}+\frac{\left(C+4B\right)x+\left(D-5B\right)}{\left(x^{2}-4x+5\right)^{2}}$$

$$x^{3}+3x^{2}+x=\frac{A}{\left(x+2\right)}\left(x+2\right)\left(x^{2}-4x+5\right)^{2}+\frac{B}{\left(x^{2}-4x+5\right)}\left(x+2\right)\left(x^{2}-4x+5\right)^{2}+\frac{\left(C+4B\right)x+\left(D-5B\right)}{\left(x^{2}-4x+5\right)^{2}}\left(x+2\right)\left(x^{2}-4x+5\right)^{2}$$

$$x^{3}+3x^{2}+x=A\left(x^{2}-4x+5\right)^{2}+B\left(x+2\right)\left(x^{2}-4x+5\right)+\left(\left(C+4B\right)x+\left(D-5B\right)\right)\left(x+2\right)$$

$$x^{3}+3x^{2}+x=$$

$$\left(Ax^{4}-4Ax^{3}+5Ax^{2}-4Ax^{3}+16Ax^{2}-20Ax+5Ax^{2}-20Ax+25A\right)+B\left(x+2\right)\left(x^{2}-4x+5\right)+\left(\left(C+4B\right)x+\left(D-5B\right)\right)\left(x+2\right)$$

$$=\left(Ax^{4}-8Ax^{3}+26Ax^{2}-40Ax+25A\right)+B\left(x^{3}-4x^{2}+5x+2x^{2}-8x+10\right)+\left(\left(C+4B\right)x^{2}+\left(D-5B\right)x+2\left(C+4B\right)x+2\left(D-5B\right)\right)$$

$$=\left(Ax^{4}-8Ax^{3}+26Ax^{2}-40Ax+25A\right)+B\left(x^{3}-2x^{2}-3x+10\right)+\left(\left(C+4B\right)x^{2}+\left(D-5B\right)x+2\left(C+4B\right)x+2\left(D-5B\right)\right)$$

$$=Ax^{4}-8Ax^{3}+Bx^{3}+26Ax^{2}-2Bx^{2}+\left(C+4B\right)x^{2}-40Ax-3Bx+\left(D-5B\right)x+2\left(C+4B\right)x+25A+10B+2\left(D-5B\right)$$

$$=Ax^{4}+\left(-8A+B\right)x^{3}+\left(26A-2B+C+4B\right)x^{2}+\left(-40A-3B+D-5B+2C+8B\right)x+25A+10B+2D-10B$$

$$x^{3}+3x^{2}+x=Ax^{4}+\left(-8A+B\right)x^{3}+\left(26A+C+2B\right)x^{2}+\left(-40A+D+2C\right)x+25A+2D$$

$$1=-8A+B 3=26A+C+2B 1=-40A+D+2C 0=25A+2D$$

$$B=1+8A 3=26A+C+2B 1=-40A+D+2C D=-\frac{25}{2}A$$

$$B=1+8A 3=26A+C+2\left(1+8A\right) 1=-40A-\frac{25}{2}A+2C D=-\frac{25}{2}A$$

$$B=1+8A 1=42A+C 1=-\frac{105}{2}A+2C D=-\frac{25}{2}A$$

$$B=1+8A C=1-42A 1=-\frac{105}{2}A+2C D=-\frac{25}{2}A$$

$$B=1+8A C=1-42A 1=-\frac{105}{2}A+2\left(1-42A\right) D=-\frac{25}{2}A$$

$$B=1+8A C=1-42A 1=-\frac{105}{2}A+2-84A D=-\frac{25}{2}A$$

This system is consistent

In partial fractions there is a general formula that says that one should split partial fractions the way that they do below. Here we have defined that P(x) and Q(x) are polynoms:

$$\frac{P(x)}{Q(x)}=\frac{polynomial}{polynomial}$$

Where the power of P<power of Q

We rewrite Q(x) to



Then we perform the partial fraction:



Where



in the partial fraction has the shape



And



has the shape:



Since this is the normal way to split into partial fractions is there a proof that shows that this partial fraction method always gives a consistent equation system that always has a solution as my partial fraction attempt number two above?