(a) If $\phi = \dfrac{\mathbf{r}.\mathbf{k}}{r^3}$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and r is the magnitude of $\mathbf{r}$, prove that

$$\nabla\phi = \frac{1}{r^5}\left[r^2\mathbf{k} - 3(\mathbf{r}.\mathbf{k})\mathbf{r}\right]$$

(b) The vector $\mathbf{v}$ is defined by $\mathbf{v} = (\mathbf{a}.\mathbf{r})\mathbf{r}$ where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{a}$ is a constant vector.

Show that $\nabla.\mathbf{v} = 4(\mathbf{a}.\mathbf{r})$ and $\nabla \times \mathbf{v} = \mathbf{a} \times \mathbf{r}$.