

You are going to approximate the function $f(x) = \sqrt{x+1}$ using the nodes $x_0 = 1$, $x_1 = 1.5$, $x_2 = 2.0$, and $x_3 = 3$. The values of f that you need are $f(1) = 1.41421$, $f(1.5) = 1.58114$, $f(2.0) = 1.73205$, and $f(3) = 2.0$.

1. Determine the $L_{n,k}$ polynomials for this set of nodes. Write each of these polynomials in standard form, *i.e.*, $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$.
2. Determine the Lagrange interpolating polynomial $P(x)$ for the function $f(x)$ with this set of nodes and write this polynomial in standard form. Use $P(x)$ to approximate $f(2.5)$, and $f(0.8)$.
3. Estimate the maximum error that could occur if $P(x)$ is used to approximate $f(x)$ for $x = 2.5$ and $x = 0.8$. Estimate the maximum error that could occur if $P(x)$ is used to approximate $f(x)$ for any x between 1 and 3. Do not compute $f(x)$.
4. Use Neville's iterated interpolation to approximate $f(2.5)$ and $f(0.8)$. Write out each complete interpolation table.