1. From the top of Mt Washington, which is 6288 feet above sea level, how far is it to the horizon? Assume that the Earth has a 3960-mile radius (one mile is 5280 feet), and give your answer to the nearest mile.

2. In mathematical discussion, a *right prism* is defined to be a solid figure that has two parallel, congruent polygonal *bases*, and rectangular *lateral faces*. How would you find the volume of such a figure? Explain your method.

3. A chocolate company has a new candy bar in the shape of a prism whose base is an 1-inch equilateral triangle and whose sides are rectangles that measure 1 inch by 2 inches. These prisms are to be packed into a box that has a regular hexagonal base with 2-inch edges, and rectangular sides that are 6 inches tall. How many candy bars can be placed in such a box?

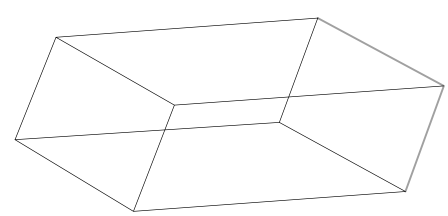
4. (Continuation) The same company also markets a rectangular chocolate bar that measures 1 cm by 2 cm by 4 cm. How many of these bars can be packed in a rectangular box that measures 8 cm by 12 cm by 12 cm? How many of these bars can be packed in rectangular box that measures 8 cm by 5 cm by 5 cm?

5. Starting at the same spot on a circular track that is 80 meters in diameter, Sandy and Candy run in opposite directions, at 300 meters per minute and 240 meters per minute, respectively. They run for 50 minutes. What distance separates Sandy and Candy when they finish? There is more than one way to interpret the word *distance* in this question.

6. Choose a positive number q (*Greek* "theta") less than 90.0 and ask your calculator for $\sin q$ and $\cos q$. Square each of these numbers, and add them. Could you have predicted the sum?

7. In the middle of the nineteenth century, octagonal barns and sheds (and even some houses) became popular. How many cubic feet of grain would an octagonal barn hold if it were 12 feet tall and had a regular base with 10-foot edges?

8. Most playing cards measure 2.25 inches by 3.5 inches. A full deck of fifty-two cards is 0.75 inches high. What is the volume of a deck of cards? If the cards were uniformly shifted so that the deck looked like the illustration at right, would the volume be affected?



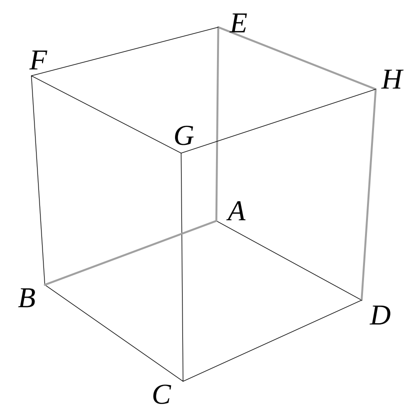
9. Build a sugar-cube pyramid in the following fashion: First make a $5\times5\times1$ bottom layer. Then center a $4\times4\times1$ layer on the first layer, center a $3\times3\times1$ layer on the second layer, and center a $2\times2\times1$ layer on the third layer. The fifth layer is a single $1\times1\times1$ cube. Express the volume of this pyramid as a percentage of the volume of a $5\times5\times5$ cube.

10. (Continuation) Repeat the construction of a sugar-cube pyramid, starting with a $10\times10\times1$ base, the dimensions of each square decreasing by one unit per layer. Using your calculator, express the volume of the pyramid as a percentage of the volume of a $10\times10\times10$ cube. Repeat, using a $20\times20\times1$ base, a $50\times50\times1$ base, and a $100\times100\times1$ base. See the pattern?

1. A vector **v** of length 6 makes a 150-degree angle with the vector [1,0], when they are placed *tail-to-tail*. Find the components of **v**.

2. Why might an Earthling believe that the Sun and the Moon are the same size?

3. Given that *ABCDEFGH* is a cube (shown at right), what is significant about the square pyramids *ADHEG*, *ABCDG*, and *ABFEG*?

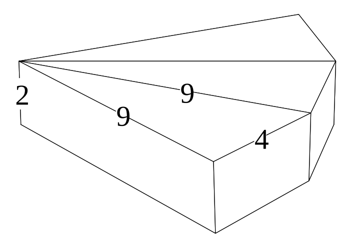


4. To the nearest tenth of a degree, find the size of the angle formed by placing the vectors [4,0] and [–6,5] tail-to-tail at the origin.

5. The angle formed by placing the vectors [4,0] and [*a*,*b*] tail-to-tail at the origin is 124 degrees. The length of [*a*,*b*] is 12. Find *a* and *b*.

6. Flying at an altitude of 29400 feet one clear day, Cameron looked out the window of the airplane and wondered how far it was to the horizon. Rounding your answer to the nearest mile, answer Cameron's question.

7. A wedge of cheese is 2 inches tall. The triangular base of this right prism has two 9-inch edges and a 4-inch edge. Several congruent wedges are arranged around a common 2-inch segment, as shown. How many wedges does it take to complete this wheel? What is the volume of the wheel, to the nearest cubic inch?



8. The Great Pyramid at Giza was originally 483 feet tall, and it had a square base that was 756 feet on a side. It was built from rectangular stone blocks measuring 7 feet by 7 feet by 15 feet. Each such block weighs seventy tons. How many tons of stone did it take to build the Great Pyramid? The volume of any pyramid is one third the product of the base area and the height.

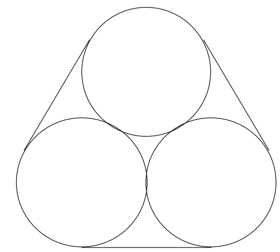
9. Although it may seem like a strange request, ask your calculator for sine and cosine values for a 120-degree angle. Try to make sense of the answers.

10. The base of pyramid *TABCD* is a 20-cm square *ABCD*. All the edges that meet at *T* are 27 cm long. Make a diagram of *TABCD*, showing *F*, the point of *ABCD* that is closest to *T*. To the nearest tenth of a cm, find the height *TF*. Find the volume of *TABCD*, to the nearest cc.

11. (Continuation) Let *P* be a point on edge *AB*, and consider the possible sizes of angle *TPF*. What position for *P* makes this angle as small as it can be? How do you know?

12. (Continuation) Let *K*, *L*, *M*, and *N* be the points on *TA*, *TB*, *TC*, and *TD*, respectively, that are 18 cm from *T*. What can be said about polygon *KLMN*? Explain.

1. The figure shows three circular pipes, all with 12-inch diameters, that are strapped together by a metal band. How long is the band?



2. (Continuation) Suppose that *four* pipes are strapped together with a snugly-fitting metal band. How long is the band?

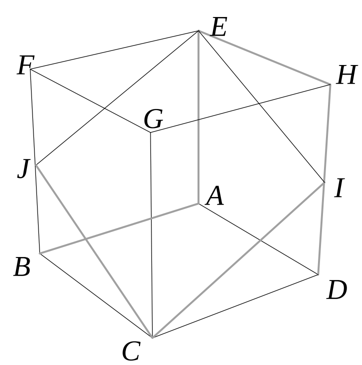
3. Which point on the circle $x^2 + y^2 - 12x - 4y = 50$ is closest to the origin? Which point is furthest from the origin? Justify your answers.

4. An isosceles triangle has two sides of length p and one of length m. In terms of these lengths, write *calculator-ready* formulas for the sizes of the angles of this triangle.

5. The lateral edges of a regular hexagonal pyramid are all 20 cm long, and the base edges are all 16 cm long. To the nearest cc, what is the volume of this pyramid? To the nearest square cm, what is the combined area of the base and six lateral faces?

6. There are two circles that go through (9,2) and that are tangent to both coordinate axes. Find their centers and their radii. Start by drawing a clear diagram.

7. The figure at right shows a $2 \times 2 \times 2$ cube *ABCDEFGH*, as well as midpoints *I* and *J* of its edges *DH* and *BF*. It so happens that *C*, *I*, *E*, and *J* all lie in a plane. Can you justify this statement? What kind of figure is quadrilateral *CIEJ*, and what is its *area*? Is it possible to obtain a polygon with a larger area by slicing the cube with a different plane? If so, show how to do it. If not, explain why it is not possible.



8. Some Exonians bought a circular pizza for \$10.80. Pat's share was \$2.25. What was the central angle of Pat's slice?

9. A wheel of radius one foot is placed so that its center is at the origin, and a paint spot on the rim is at (1,0). The wheel is spun q degrees in a counterclockwise direction. Now what are the coordinates of the paint spot?

10. A plot of land is bounded by a 140-degree circular arc and two 80-foot radii of that circle. To the nearest foot, find the *perimeter* of the plot. Find its area, to the nearest square foot.

11. Deniz observes that the Sun can barely be covered by holding an aspirin tablet at arm's length and closing one eye. The aspirin is 7 mm in diameter, and it is being held 80 cm from Deniz's eye. Find the *apparent size* of the Sun, which is the size of the angle *subtended* by the Sun.

12. Circles centered at *A* and *B* are tangent at *T*. Explain why *A*, *T*, and *B* must be collinear.

13. At constant speed, a wheel rotates once counterclockwise every 10 seconds. Its center is (0,0) and its radius is 1 foot. A paint spot is initially at (1,0); where is it *t* seconds later?

1. The base of a pyramid is the regular polygon *ABCDEFGH*, which has 14-inch sides. All eight of the pyramid's lateral edges, *VA*, *VB*, etc, are 25 inches long. To the nearest tenth of an inch, calculate the height of pyramid *VABCDEFGH*.

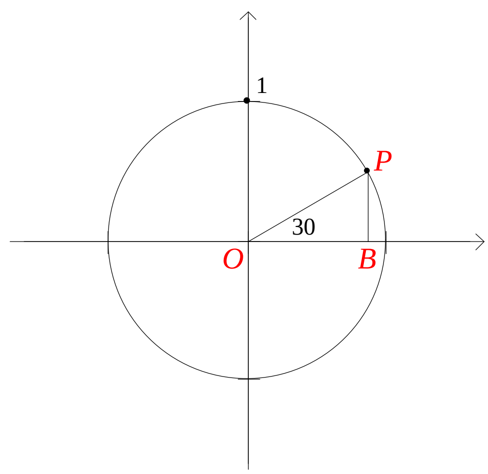
2. (Continuation) To the nearest tenth of a degree, calculate the size of the dihedral angle formed by the octagonal base and the triangular face *VAB*.

3. (Continuation) Points *A'*, *B'*, *C'*, *D'*, *E'*, *F'*, *G'*, and *H'* are marked on edges *VA*, *VB*, *VC*, *VD*, *VE*, *VF*, *VG*, and *VH*, respectively, so that the segments *VA'*, *VB'*, etc are all 20 inches long. Find the volume ratio of pyramid *VA'B'C'D'E'F'G'H'* to pyramid *VABCDEFGH*. Find the volume ratio of *frustum A'B'C'D'E'F'G'H'ABCDEFGH* to pyramid *VABCDEFGH*.

4. Quinn is running around the circular track $x^2 + y^2 = 10000$, whose radius is 100 meters, at 4 meters per second. Quinn starts at the point $(100,0)$ and runs in the counterclockwise direction. After 30 minutes of running, what are Quinn's coordinates?

5. (a) The hypotenuse of a right triangle is 1000.000, and one of its angles is 87.000 degrees. Find the legs and the area of the triangle, correct to three decimal places.
(b) Write a formula for the area of a right triangle in which *h* is the length of the hypotenuse and *A* is the size of one of the acute angles.
(c) Apply your formula (b) to redo part (a). Did you get the same answer? Explain.

6. Consider the *unit circle*, whose center is the origin *O* and whose radius is 1. Suppose that *P* is on the circle in the first quadrant, and that the angle formed by segment *OP* and the positive *x*-axis is $30°$. Mark *B* on the *x*-axis so that angle *OBP* is right. Find *PB* and *OB*, expressed in exact form. What are the *x*- and *y*-coordinates of *P*? Express the *x*- and *y*-coordinates of *P* using trigonometric functions of a 30-degree angle.



7. (Continuation) Repeat the preceding, assuming now that *OP* makes a $60°$ angle with the positive *x*-axis.

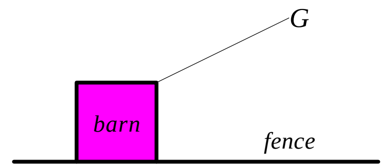
8. (Continuation) Repeat the preceding, assuming that *OP* makes a $150°$ angle with the positive *x*-axis. Notice that *B* is left of the *y*-axis. Explain why triangle *OBP* is congruent to the previous triangles *OBP*. Find coordinates (*x*,*y*) for *P*. If it were up to you to give a meaning to the expression $\cos 150$, what would it be? Does your calculator agree with your choice?

9. Find the center and the radius for each of the circles $x^2 - 2x + y^2 - 4y - 4 = 0$ and $x^2 - 2x + y^2 - 4y + 5 = 0$. How many points fit the equation $x^2 - 2x + y^2 - 4y + 9 = 0$?

10. What is the result of graphing the equation $(x-h)^2 + (y-k)^2 = r^2$?

1. A *half-turn* is a 180-degree rotation. Apply the half-turn centered at (3,2) to the point (7,1). Find coordinates of the image point. Find coordinates for the image of (x,y).

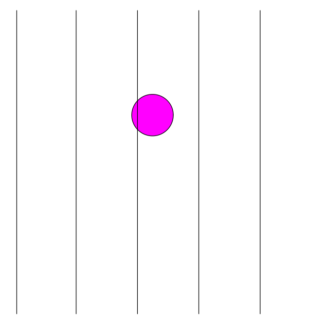
2. Find the total grazing area of the goat G represented in the figure (a top view) shown at right. The animal is tied to a corner of a 40'-by-40' barn, by an 80' rope. One of the sides of the barn is extended by a fence. Assume that there is grass everywhere except inside the barn.



3. A 16.0-inch chord is drawn in a circle whose radius is 10.0 inches. What is the *angular size* of the minor arc of this chord? What is the length of the arc, to the nearest tenth of an inch?

4. What graph is traced by the parametric equation $(x, y) = (\cos q, \sin q)$?

5. What is the area enclosed by a circular *sector* whose radius is r and whose arc length is s?



6. A coin with a 2-cm diameter is dropped onto a sheet of paper ruled by parallel lines that are 3 cm apart. Which is more likely, that the coin will land on a line, or that it will not?

7. A wheel whose radius is 1 is placed so that its center is at (3,2). A paint spot on the rim is found at (4,2). The wheel is spun q degrees in the counterclockwise direction. Now what are the coordinates of that paint spot?

8. A 36-degree counterclockwise rotation centered at the origin sends the point $A = (6,3)$ to the image point A'. To three decimal places, find coordinates for A'.

9. In navigational terms, a *minute* is one sixtieth of a *degree*, and a *second* is one sixtieth of a minute. To the nearest foot, what is the length of a one-second arc on the Equator? The radius of the Earth is 3960 miles.

10. A sector of a circle is enclosed by two 12.0-inch radii and a 9.0-inch arc. Its perimeter is therefore 33.0 inches. What is the area of this sector, to the nearest tenth of a square inch? What is the central angle of the sector, to the nearest tenth of a degree?

11. (Continuation) There is another circular sector — part of a circle of a different size — that has the same 33-inch perimeter and that encloses the same area. Find its central angle, radius, and arc length, rounding the lengths to the nearest tenth of an inch.

12. Use the unit circle to find $\sin 240$ and $\cos 240$, without using a calculator. Then use your calculator to check your answers. Notice that your calculator expects you to put parentheses around the 240, which is because sin and cos are functions. Except in cases where the parentheses are required for clarity, they are often left out.

1. Given that $\cos 80 = 0.173648\ldots$, explain how to find $\cos 100$, $\cos 260$, $\cos 280$, and $\sin 190$ without using a calculator.

2. Use the unit circle to define $\cos\theta$ and $\sin\theta$ for any number θ between 0 and 360, inclusive. Then explain how to use $\cos\theta$ and $\sin\theta$ to define $\tan\theta$.

3. Show that your method in the previous question allows you to define $\cos\theta$, $\sin\theta$, and $\tan\theta$ for numbers θ greater than 360 and for numbers θ less than 0, too. What do you suppose it means for an angle to be *negative*?

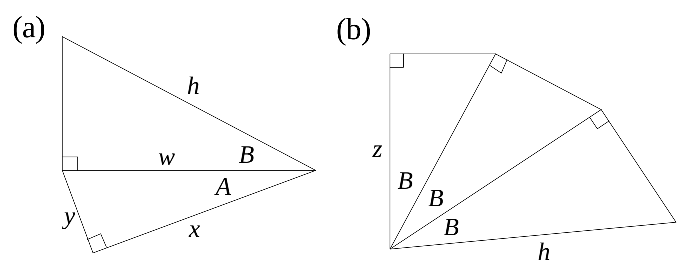
4. A half-turn centered at $(-3,4)$ is applied to $(-5,1)$. Find coordinates for the image point. What are the coordinates when the half-turn centered at (a,b) is applied to (x,y)?

5. Translate the circle $x^2 + y^2 = 49$ by the vector $[3,-5]$. Write an equation for the image circle.

6. Point by point, a dilation transforms the circle $x^2 - 6x + y^2 - 8y = -24$ onto the circle $x^2 - 14x + y^2 - 4y = -44$. Find the center and the magnification factor of this transformation.

7. (Continuation) The circles have two common external tangent lines, which meet at the dilation center. Find the size of the angle formed by these lines, and write an equation for each line.

8. Using the figures at right, express the lengths *w, x, y*, and *z* in terms of length *h* and angles *A* and *B*.



9. Find at least two values for θ that fit the equation $\sin\theta = \frac{1}{2}\sqrt{3}$. How many such values are there?

10. Choose an angle θ and calculate $(\cos\theta)^2 + (\sin\theta)^2$. Repeat with several other values of θ. Explain the result. *Note*: It is customary to write $\cos^2\theta + \sin^2\theta$ instead of $(\cos\theta)^2 + (\sin\theta)^2$

11. A 15-degree counterclockwise rotation centered at (2,1) sends (4,6) to another point (*x*,*y*). Find *x* and *y*, correct to three decimal places.

12. What graph is traced by the parametric equation $(x,y) = (2+\cos t, 1+\sin t)$?

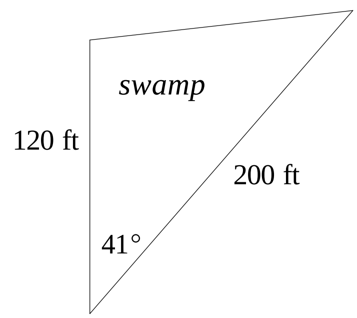
13. A circle centered at the origin meets the line $-7x + 24y = 625$ tangentially. Find coordinates for the point of tangency.

14. Write without parentheses: (a) $(xy)^2$ (b) $(x+y)^2$ (c) $(a \sin B)^2$ (d) $(a + \sin B)^2$

1. A transformation T is defined by $\mathsf{T}(x, y) = (2,7) - [x - 2, y - 7]$. An equivalent definition is $\mathsf{T}(x, y) = (4 - x, 14 - y)$. Use the first definition to explain what kind of transformation T is.

2. Use the unit circle to help you find all solutions q between 0 and 360, inclusive:
(a) $\cos q = -1$ (b) $\cos q = 0.3420$ (c) $\sin q = -\frac{1}{2}\sqrt{2}$ (d) $\tan q = 6.3138$

3. A triangular plot of land has the SAS description indicated in the figure at right. Although a swamp in the middle of the plot makes it awkward to *measure* any of the altitudes of this triangle, at least one of them can be *calculated*. Show how. Then use your answer to find the area of the triangle, to the nearest square foot.



4. (Continuation) Find the length of the third side of the triangle, to the nearest foot.

5. A 15-degree counterclockwise rotation about (4,6) transforms (2,1) onto another point (*x*,*y*). Find *x* and *y*, correct to three decimal places.

6. Using the line $y = x$ as a mirror, find the reflected image of the point (a, b). What are the coordinates of the point on the line that is closest to (a, b)?

7. A sector has radius *r* and central angle q. Write formulas for its arc length and perimeter.

8. A bird flies linearly, according to the equation $(x, y, z) = (5,6,7) + t[2,3,1]$. Assume that the Sun is directly overhead, making the Sun's rays perpendicular to the *xy*-plane which represents the ground. The bird's shadow is said to be *projected* perpendicularly onto the ground. Find an equation that describes the motion of the shadow.

9. A coin of radius 1 cm is tossed onto a plane surface that has been *tesselated* (tiled) by rectangles whose measurements are all 8 cm by 15 cm. What is the probability that the coin lands within one of the rectangles?

10. What graph is traced by the parametric equation $(x, y) = (3\cos t, 3\sin t)$? What about the equation $(x, y) = (7 + 3\cos t, -2 + 3\sin t)$?

11. A *quarter-turn* is a 90-degree rotation. If the counterclockwise quarter-turn centered at (3,2) is applied to (7,1), what are the coordinates of the image? What are the image coordinates when this transformation is applied to a general point (*x*,*y*)?

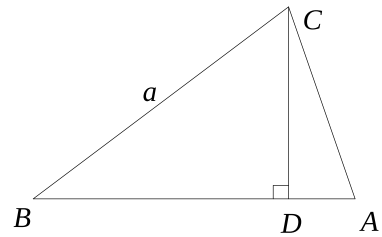
12. Suppose that the lateral faces *VAB*, *VBC*, and *VCA* of triangular pyramid *VABC* all have the same height drawn from *V*. Let *F* be the point in base *ABC* that is closest to *V*, so that *VF* is the altitude of the pyramid. Show that *F* is one of the special points of triangle *ABC*.

1. Simplify: (a) $x\cos^2 q + x\sin^2 q$ (b) $x\cos^2 q + x\cos^2 q + 2x\sin^2 q$

2. A 12.0-cm segment makes a 72.0-degree angle with a 16.0-cm segment. To the nearest tenth of a cm, find the third side of the triangle determined by this SAS information.

3. (Continuation) Find the area of the triangle, to the nearest square centimeter.

4. In the diagram at right, CD is the altitude from C.
(a) Express CD in terms of angle B and side a.
(b) Express BD in terms of angle B and side a.
(c) Simplify the expression $(a\sin B)^2 + (a\cos B)^2$ and discuss its relevance to the diagram.
(d) Why was $a\sin B$ used instead of $\sin B \cdot a$?



5. A 12.0-centimeter segment makes a 108.0-degree angle with a 16.0-centimeter segment. To the nearest millimeter, find the third side of the triangle determined by this SAS information.

6. (Continuation) Find the area of the triangle, to the nearest square centimeter.

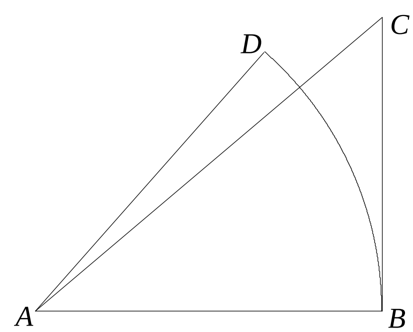
7. Aomawa has made some glass prisms to be sold as window decorations. Each prism is four inches tall, and has a regular hexagonal base with half-inch sides. They are to be shipped in cylindrical tubes that are 4 inches tall. What radius should Aomawa use for the tubes? Once a prism is inserted into its tube, what volume remains for packing material?

8. Find all solutions t between 360 and 720, inclusive:
(a) $\cos t = \sin t$ (b) $\tan t = -4.3315$ (c) $\sin t = -0.9397$ (d) $\cos t < \frac{1}{2}\sqrt{3}$

9. Find the center and the radius of the circle $x^2 + y^2 - 2ax + 4by = 0$

10. The wheels of a moving bicycle have 27-inch diameters, and they are spinning at 200 revolutions per minute. How fast is the bicycle traveling, in miles per hour? Through how many degrees does a wheel turn each second?

11. In the figure at right, arc BD is centered at A, and it has the same length as tangent segment BC. Explain why sector ABD has the same area as triangle ABC.



12. Find all solutions A between 0 and 360:
(a) $\cos A = \cos 251$ (b) $\cos A = 1.5$
(c) $\sin A = \sin 220$ (d) $\cos A = \cos(-110)$

13. Does every equation of the form $x^2 + mx + y^2 + ny = p$ represent a circle? Explain.

1. Consider the transformation $\mathsf{T}(x, y) = \left(\frac{4}{5}x - \frac{3}{5}y, \frac{3}{5}x + \frac{4}{5}y\right)$, which is a rotation centered at the origin. Describe the sequence of points that arise when T is applied repeatedly, starting with the point $A_0 = (5,0)$. In other words, A_1 is obtained by applying T to A_0, then A_2 is obtained by applying T to A_1, and so forth. Be as accurate as you can in your description.

2. How long is the shadow cast on the ground (represented by the the *xy*-plane) by a pole that is seven meters tall, given that the Sun's rays are parallel to the vector $[5,3,-2]$?

3. A conical cup has a 10-cm diameter and is 12 cm deep. How much water can this cup hold?

4. (Continuation) The water in the cup is 6 cm deep. What percentage of the cup is filled?

5. (Continuation) Dana takes a paper cone of the given dimensions, cuts it along a straight line from the rim to the vertex, then flattens the paper out on a table. Find the radius, the arc length, and the central angle of the resulting circular sector.

6. A javelin lands with six feet of its length sticking out of the ground, making a 52-degree angle with the ground. The Sun is directly overhead. The javelin's shadow on the ground is an example of a *projection*. Find its length, to the nearest inch.

7. The *dot product* of vectors **u** = [*a*,*b*] and **v** = [*m*,*n*] is the number $\mathbf{u} \bullet \mathbf{v} = am + bn$. The *dot product* of vectors **u** = [*a*,*b*,*c*] and **v** = [*m*,*n*,*p*] is the number $\mathbf{u} \bullet \mathbf{v} = am + bn + cp$. In general, the dot product of two vectors is the sum of all the products of corresponding components. Let $\mathbf{u} = [-2,3,1]$, $\mathbf{v} = [0,1,2]$, and $\mathbf{w} = [1,2-1]$. Calculate

   (a) $4\mathbf{u}$ (b) $\mathbf{u} + \mathbf{v}$ (c) $4\mathbf{u} - \mathbf{v}$ (d) $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w})$ (e) $\mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$

8. In triangle *ABC*, it is given that *BC* = 7, *AB* = 3, and $\cos B = \frac{11}{14}$. Find the length of the projection of *AB* onto *BC* and the length of the projection of *BC* onto *AB*.

9. Find the coordinates of the shadow cast on the *xy*-plane by a small object placed at the point $(10,7,20)$, assuming that the Sun's rays are parallel to the vector $[5,3,-2]$.

10. Allie is riding on a merry-go-round, which has a 25-foot radius and which is turning at 36 degrees per second. Seeing a friend in the crowd, Allie steps off the outer edge of the merry-go-round and suddenly finds it necessary to start running. At what speed, in miles per hour?

11. A coin of radius 1 cm is tossed onto a plane surface that has been tesselated by right triangles whose sides have lengths 8 cm, 15 cm, and 17 cm. What is the probability that the coin lands within one of the triangles?

12. A circular sector has a 8.26-inch radius and a 12.84-inch arc length. There is another sector that has the same area and the same perimeter. What are its measurements?

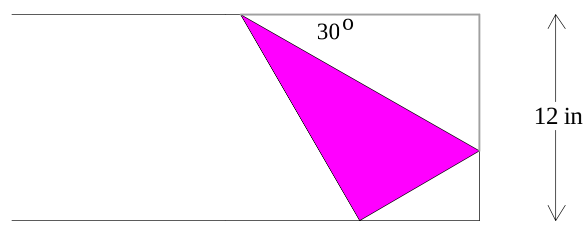
13. (Continuation) Given a circular sector, is there always a different sector that has the same area and the same perimeter? Explain your answer.

1. Solve for *y*: $x^2 = a^2 + b^2 - 2aby$

2. A segment that is *a* units long makes a *C*-degree angle with a segment that is *b* units long. In terms of *a*, *b*, and *C*, find the third side of the triangle defined by this SAS description. You have done numerical versions of this question. Start by finding the length of the altitude drawn to side *b*, as well as the length of the perpendicular projection of side *a* onto side *b*. The resulting formula is known as the *Law of Cosines*.

3. (Continuation) What is the area of the triangle defined by *a*, *b*, and *C*?

4. The figure at right shows a long rectangular strip of paper, one corner of which has been folded over to meet the opposite edge, thereby creating a 30-degree angle. Given that the width of the strip is 12 inches, find the length of the crease.



5. (Continuation) Suppose that the size of the folding angle is q degrees. Use trigonometry to express the length of the crease as a function of q. Check using the case $q = 30$.

6. (Continuation) Find approximately that value of q that makes the crease as short as it can be. Restrict your attention to angles that are smaller than 45 degrees. (Why is this necessary?)

7. A triangle has an 8-inch side, a 10-inch side, and an area of 16 square inches. What can you deduce about the angle formed by these two sides?

8. Jamie rides a *Ferris wheel* for five minutes. The diameter of the wheel is 10 meters, and its center is 6 meters above the ground. Each revolution of the wheel takes 30 seconds. Being more than 8 meters above the ground causes Jamie to suffer an anxiety attack. For how many seconds does Jamie feel uncomfortable?

9. (Continuation) What graph is traced by the equation $(x, y) = (5\sin 12t, 6 - 5\cos 12t)$? Think of another equation that will produce the same graph. Use your calculator to check.

10. Find two different parametric descriptions for the circle of radius 4 centered at $(-3,2)$.

11. Let **u** = [*a*,*b*,*c*], **v** = [*p*,*q*,*r*], and **w** = [*k*,*m*,*n*] for the following questions:
   (a) Verify that $\mathbf{u} \bullet \mathbf{v}$ is the same number as $\mathbf{v} \bullet \mathbf{u}$, for any vectors **u** and **v**.
   (b) What is the significance of the number $\mathbf{u} \bullet \mathbf{u}$?
   (c) What does the equation $\mathbf{u} \bullet \mathbf{v} = 0$ tell us about the vectors **u** and **v**?
   (d) Is it true that $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$ holds for all vectors **u**, **v**, and **w**?
   (e) If **u** and **v** represent the sides of a parallelogram, then **u** + **v** and **u** – **v** represent the diagonals. Justify this, then explain what the equation $(\mathbf{u} + \mathbf{v}) \bullet (\mathbf{u} - \mathbf{v}) = 0$ tells us about the parallelogram. Give an example of two nonzero vectors **u** and **v** that fit this equation.

1. Dana takes a sheet of paper, cuts a 120-degree circular sector from it, then rolls it up and tapes the straight edges together to form a cone. Given that the sector radius is 12 cm, find the height and volume of this paper cone.

2. Find the third side of a triangle in which a 4.00-inch side and a 6.00-inch side are known to make a 56.0-degree angle. Round your answer to three significant digits.

3. Find all solutions w between 0 and 360, inclusive:
   (a) $\cos w = \cos(-340)$ (b) $\cos w = \sin 20$ (c) $\sin w = \cos(-10)$
   (d) $\sin w < -\frac{1}{2}$ (e) $1 < \tan w$

4. The radius of the circumscribed circle of the triangle ABC is 15 cm. Given that B is a 49-degree angle, find the length of side AC.

5. (Continuation) The radius of the circumscribed circle of the triangle ABC is r cm. Given that B is a q -degree angle, find the length of side AC, in terms of r and q .

6. A counterclockwise quarter-turn Q about the origin is applied to the point (x,y). What are the coordinates of the image point? Your answer will take the form $\mathsf{Q}(x, y) = (ax + by, cx + dy)$.

7. The perimeter of a triangle, its area, and the radius of the circle inscribed in the triangle are related in an interesting way. Prove that the radius of the circle times the perimeter of the triangle equals twice the area of the triangle.

8. The table at right shows some measurements made on a circle with a one-meter radius. Every entry in the s-column stands for the length of an arc, and the corresponding entry in the c-column stands for the length of the chord of that arc, both measured in meters. Explain why $c < s$, and then determine the range of values for c and for s. With s on the horizontal axis and c on the vertical axis, sketch an approximate graph of c vs s.

s	c
0.256	0.255
0.618	0.608
1.234	1.157
1.571	1.414
1.896	1.625

9. (Continuation) Let q be the central angle that intercepts s and c. Write equations for s and c in terms of q , then combine them to express c as a function of s. Graph this relationship.

10. Given a vector **u**, the familiar *absolute-value* notation $|\mathbf{u}|$ is often used for its *magnitude*. Thus the expressions $\mathbf{u} \bullet \mathbf{u}$ and $|\mathbf{u}|^2$ both mean the same thing. What exactly *do* they mean?

11. A familiar algebraic result is that, for any two numbers a and b, the product of $a - b$ times itself is equal to $a^2 - 2ab + b^2$. Does the analogous result hold for dot products of a vector $\mathbf{u} - \mathbf{v}$ with itself? In other words, is it true that $(\mathbf{u}-\mathbf{v}) \bullet (\mathbf{u}-\mathbf{v}) = \mathbf{u} \bullet \mathbf{u} - 2\mathbf{u} \bullet \mathbf{v} + \mathbf{v} \bullet \mathbf{v}$? Justify your conclusion, trying not to express vectors **u** and **v** in component form.

12. A triangle has a 56-degree angle, formed by a 10-inch side and an x-inch side. Given that the area of the triangle is 18 square inches, find x.

1. Devon's bike, which has wheels that are 27 inches in diameter, picks up a tack in the front wheel. Devon rolls another 100 feet and stops the bike. How far above the ground is the tack?

2. An isosceles triangle has two 10.0-inch sides and a $2p$-inch side. Find the radius of the inscribed circle of this triangle, in the cases $p = 5.00$, $p = 6.00$, and $p = 8.00$.

3. (Continuation) Write an expression for the inscribed radius r in terms of the variable p, then find the value of p, to the nearest hundredth, that gives the maximum value of r.

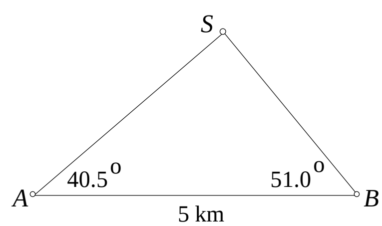
4. Triangle *ABC* has a 63.0-degree angle at *B*, and side *AC* is 13.6 cm long. What is the diameter of the circle circumscribed about *ABC*?

5. (Continuation) Given any triangle *ABC*, with sides a, b, and c opposite angles *A*, *B*, and *C*, respectively, what can be said about the three ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$? This result is known as the *Law of Sines*.

6. Let $\mathbf{u} = [3,-4]$ and $\mathbf{v} = [8,15]$. Calculate the four numbers $|\mathbf{u}|$, $|\mathbf{v}|$, $|\mathbf{u}-\mathbf{v}|$, and $|\mathbf{u}|-|\mathbf{v}|$. Do the same with the vectors $\mathbf{u} = [2,6,-3]$ and $\mathbf{v} = [2,2,1]$.

7. The base radius of a cone is 6 inches, and the cone is 8 inches tall. To the nearest square inch, what is the area of the lateral surface of the cone?

8. Two observers who are 5 km apart simultaneously sight a small airplane flying between them. One observer measures a 51.0-degree inclination angle, while the other observer measures a 40.5-degree inclination angle. At what altitude is the airplane flying?



9. If a triangle has sides of lengths a and b, which make a C-degree angle, then the length of the side opposite C is c, where $c^2 = a^2 + b^2 - 2ab\cos C$. This is the SAS version of the Law of Cosines. Explain the terminology. Derive an equivalent SSS version of the Law of Cosines, which gives the cosine of the angle in terms of the lengths of the three sides. Now use it to find the angles of the triangle whose sides have lengths 4 cm, 5 cm, and 6 cm.

10. What is the length of the vector $[5\cos q, 5\sin q]$? If the vector [5,0] is rotated 36 degrees in the counterclockwise direction, what are the components of the resulting vector?

11. Infinitely many different sectors can be cut from a circular piece of paper with a 12-cm radius, and any such sector can be fashioned into a paper cone with a 12-cm *slant height*.
(a) Show that the volume of the cone produced by the 180-degree sector is larger than the volume of the cone produced by the 120-degree sector.
(b) Find a sector of the same circle that will produce a cone whose volume is even larger.
(c) Express the volume of a cone formed from this circle as a function of the central angle of the sector used to form it, then find the sector that produces the cone of greatest volume.

1. Suppose that two vectors **u** and **v** fit the equation $(\mathbf{u}-\mathbf{v})\bullet(\mathbf{u}-\mathbf{v}) = \mathbf{u}\bullet\mathbf{u}+\mathbf{v}\bullet\mathbf{v}$. How must these vectors **u** and **v** be related? What familiar theorem does this equation represent?

2. A *matrix* provides a convenient way to display and process certain kinds of data. Here is a simple example: Suppose that the IOKA sold tickets to 186 adults on Friday, 109 adults on Saturday, 111 adults on Sunday, 103 children on Friday, 127 children on Saturday, 99 children on Sunday, 77 senior citizens on Friday, 67 senior citizens on Saturday, and 58 senior citizens on Sunday. This data is displayed in the 3-by-3 sales matrix **S** shown at right. The descriptive labels given in the margin allow the reader to easily remember what all the numbers mean. Invent your own example of numerical data that can be displayed like this in a rectangular array.

	Ch	*Ad*	*Sen*
Fr	103	186	77
Sa	127	109	67
Su	99	111	58

3. (Continuation) The IOKA's ticket prices can be read from the 3-by-1 matrix **P**, which is shown at right. Such a matrix is often called a *column vector*. The first row of matrix **S** is a 3-component *row vector*. What is the dot product of these two vectors? What does it *mean*? What about the dot products of **P** with the other rows of **S**?

Ch	2.50
Ad	4.50
Sen	1.75

4. (Continuation) *Matrix multiplication* is performed by calculating all possible dot products of row vectors from the first matrix and column vectors from the second matrix. How many such products can be formed by multiplying matrix **S** times matrix **P**? How would you organize them into a new matrix, called $\mathbf{S}\bullet\mathbf{P}$? What do the entries of $\mathbf{S}\bullet\mathbf{P}$ mean?

5. A triangle has a 5-inch side and an 8-inch side, which form a 60-degree angle. Find
   (a) the area of this triangle;
   (b) the length of the projection of the 8-inch side onto the 5-inch side;
   (c) the length of the third side of this triangle;
   (d) the sizes of the other two angles of this triangle;
   (e) the length of the median drawn to the 8-inch side;
   (f) the length of the bisector of the angle opposite the 8-inch side;
   (g) another triangle that has a 5-inch side, an 8-inch side, and the same area as this triangle.

6. Draw the unit circle and a first-quadrant ray from the origin that makes an angle q with the positive x-axis. Let B be the point on this ray whose x-coordinate is 1, and let A = (1,0). In terms of q, find the length of the segment AB, which is tangent to the circle. Hmm…

7. Triangle PEA has a 20-degree angle at P and a 120-degree angle at E, and the length of side EA is 6 inches. Find the lengths of the other two sides of this triangle.

8. Let **u** = [2,–3,1] and **v** = [0,1,4]. Calculate the vector **u** – **v**. Place **u** and **v** tail-to-tail to form two sides of a triangle. With regard to this triangle, what does **u** – **v** represent? Calculate the number $\mathbf{u}\bullet\mathbf{u}$ and discuss its relevance to the picture you have drawn. Finally, calculate the product $(\mathbf{u}-\mathbf{v})\bullet(\mathbf{u}-\mathbf{v})$ and explain its significance.

1. The lengths QR, RP, and PQ in triangle PQR are often denoted p, q, and r, *respectively*. What do the formulas $\frac{1}{2}pq\sin R$ and $\frac{1}{2}qr\sin P$ mean? Justify the equation $\frac{1}{2}pq\sin R = \frac{1}{2}qr\sin P$, then simplify it to a familiar form.

2. The price of a large pepperoni pizza is \$11 at New England Pizza, \$12 at Romeo's, and \$11.75 at Supreme. These shops charge \$3.50, \$3.25, and \$3.75, respectively, for a Greek salad. What would the bill be at each shop for seven pizzas and five salads for a dorm party?

3. (Continuation) What would each shop charge for two pizzas and a dozen salads? Show how this problem, as well as the previous one, can be solved by forming two suitable matrices and then multiplying them.

4. If two angles are supplementary, then their sines are equal. Explain why. What about the cosines of supplementary angles? If you are not sure, do some examples on your calculator.

5. An isosceles triangle has two sides of length w that make a 2α-degree angle. Write down two different formulas for the *area* of this triangle, in terms of w and α. By equating the formulas, discover a relation involving $\sin 2\alpha$, $\sin\alpha$, and $\cos\alpha$.

6. A parallelogram has a 7-inch side and a 9-inch side, and the longer diagonal is 14 inches long. Find the length of the other diagonal. Do you *really* need your calculator for this one?

7. Multiplying two matrices consists of calculating several dot products and then arranging them to form a new matrix. There is a natural way to arrange these dot products, each of which combines a row vector from the first matrix and a column vector from the second matrix. To test your intuition, calculate the following matrix products:

   (a) $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ -2 & 5 \end{bmatrix} \bullet \begin{bmatrix} 6 & 0 \\ -3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -3 & 6 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ 4 \\ -7 \end{bmatrix}$

8. Describe all the points on the Earth's surface that are exactly 4000 miles from the North Pole. If you need to, use 3960 miles for the radius of the Earth.

9. (a) Let $\mathbf{u} = [2,1]$ and $\mathbf{v} = [1,-3]$. Find the angle formed by $\mathbf{u}$ and $\mathbf{v}$.
   (b) Let $\mathbf{u} = [-1,0,1]$ and $\mathbf{v} = [0,2,-2]$. Find the angle formed by $\mathbf{u}$ and $\mathbf{v}$.

10. Given a triangle ABC in which angle B is exactly twice the size of angle C, *must* it be true that side AC is exactly twice the size of side AB? *Could* it be true?

11. The tires on Sasha's Volvo have 23-inch diameters, and are guaranteed to last for 40000 miles of driving. Approximately how many times will each tire turn during such a lifetime?

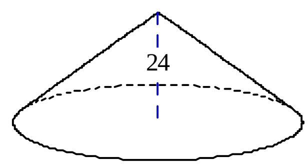
12. There are two noncongruent triangles that have a 9-inch side, a 10-inch side, and that enclose 36 square inches of area. Find the length of the third side in each of these triangles.

1. Centered 6 meters above the ground, a Ferris wheel of radius 5 meters rotates at 1 degree per second. Assuming that Jamie's ride begins at the lowest point on the wheel, find how far Jamie is above the ground after 29 seconds; after 331 seconds; after *t* seconds.

2. (Continuation) Use your calculator to graph the equation $y = 6 - 5\cos x$. What does this picture tell you about Jamie's ride? Would a graph of $y = 6 + 5\cos x$ mean anything?

3. Draw vectors **u** and **v** tail-to-tail so that they make a q -degree angle. Draw the vector $\mathbf{u} - \mathbf{v}$, the third side of the triangle, and check to see that it points in the right direction.
(a) Solve for $\cos q$ using the SSS version of the Law of Cosines, expressing all lengths in terms of **u**, **v**, and $\mathbf{u} - \mathbf{v}$.
(b) If you use vector algebra to simplify the numerator as much as possible, you will discover an interesting new result connecting $\mathbf{u} \bullet \mathbf{v}$ to $\cos q$.

4. The highway department keeps its sand in a conical storage building that is 24 feet high and 64 feet in diameter. To estimate the cost of painting the building, the lateral surface area of the cone is needed. To the nearest square foot, what *is* the area?



5. Let $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\mathbf{N} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $\mathbf{P} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Evaluate all possible two-matrix products.

6. Find the angle formed when [3,4] and [–5,12] are placed tail-to-tail, then find components for the vector that results when [3,4] is projected onto [–5,12].

7. Its center at *O* = (0,0), the unit circle $x^2 + y^2 = 1$ goes through *P* = (1,0). The line *y* = 0.6 intersects the circle at *A* and *B*, with *A* in the first quadrant. The angles *POA* and *POB* are said to be in *standard position*, because their *initial ray OP* points in the positive *x*-direction. (Their *terminal rays* are *PA* and *PB*.) Find the sizes of these angles. How are they related?

8. (Continuation) Let *D* = (–0.6,0.8). If we restrict ourselves to a single revolution, there are two angles in standard position that could be named *POD*. The one determined by minor arc *PD* is said to be *positive*, because it opens in the counterclockwise direction. Find its degree measure. The one determined by major arc *PD* is said to be *negative*, because it opens in the clockwise direction. Find its degree measure.

9. Two matrices can be multiplied only if their sizes are compatible. Suppose that **U** is an *m*-by-*n* matrix, and that **V** is a *p*-by-*q* matrix. In order for $\mathbf{U} \bullet \mathbf{V}$ to make sense, what must be true of the dimensions of these matrices? Although matrix multiplication uses dot products, it is customary to write **UV** without the dot, which will be done from now on.

10. A *sphere* consists of all the points that are 5 units from its center (2,3,–6). Write an equation that describes this sphere. Does the sphere intersect the *xy*-plane? Explain.

1. Sam owns a triangular piece of land on which the tax collector wishes to determine the correct property tax. Sam tells the collector that "the first side lies on a straight section of road and the second side is a stone wall. The wall meets the road at a 24-degree angle. A 180-foot-long fence forms the third side of the property, which meets the wall at a point that is 340 feet from the corner where the wall meets the road." After a little thought, the tax collector realizes that Sam's description of his property is ambiguous, because there are still two possible lengths for the first side. By means of a clear diagram, explain this situation, and calculate the two possible areas, to the nearest square foot.

2. Consider again the sphere of radius 5 centered at (2,3,–6). Describe the intersection of the sphere with the *xz*-plane. Write an equation (or equations) for this curve.

3. For each of the following, tell how many noncongruent triangles *PQR* fit the given description, and find the size of angle *Q* for each. Make a separate diagram for each case.
   (a) $p = 3$, $q = 5$, angle $P = 27$ degrees; (b) $p = 8$, $q = 5$, angle $P = 57$ degrees;
   (c) $p = 7$, $q = 8$, angle $P = 70$ degrees; (d) $p = 10$, $q = 20$, angle $P = 30$ degrees;

4. Describe the effect of each of the following geometric transformations. To generate and test your hypotheses, transform some simple points.
   (a) $\mathsf{T}(x, y) = (-7x, -7y)$ (b) $\mathsf{T}(x, y) = (-y, x)$
   (c) $\mathsf{T}(x, y) = (-y, -x)$ (d) $\mathsf{T}(x, y) = (0.6x - 0.8y, 0.8x + 0.6y)$

5. (Continuation) Each transformation appears in the general form $\mathsf{T}(x, y) = (ax + by, cx + dy)$, for suitable constants *a*, *b*, *c*, and *d*. For example, $a = -7$, $b = 0 = c$, and $d = -7$ in (a). What are the values of *a*, *b*, *c*, and *d* for the remaining examples?

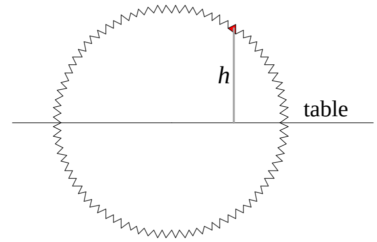
6. (Continuation) Because the expression $ax + by$ is really the dot product $[a, b] \bullet [x, y]$, it is inviting to use matrices to represent these geometric transformations. Explain the connection between the matrix product $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix}$ and the transformation $\mathsf{T}(x, y) = (ax + by, cx + dy)$. Write the *coefficient matrix* $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ for each of the transformations above.

7. Find the angle formed when [4,4,2] and [4,3,12] are placed tail-to-tail; then find the components of the vector that results when [4,3,12] is projected onto [4,4,2].

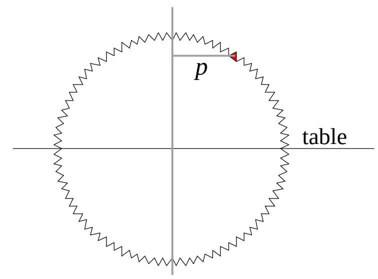
8. During one term in Math 310, Min Lee took seven tests, the last of which carried twice the weight of each of the others when averages were computed. Min's test-score vector for the term was [84,78,91,80,72,88,83]. Show that Min's final average can be calculated as a dot product of this vector with another seven-component vector. Explain how the teacher created a list of test averages for the whole class by multiplying two suitably chosen matrices.

9. What does the graph of $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ look like?

1. A large circular saw blade with a 1-foot radius is mounted so that exactly half of it shows above the table. It is spinning slowly, at one degree per second. One tooth of the blade has been painted red. This tooth is initially 0 feet above the table, and rising. What is the height after 37 seconds? After 237 seconds? After *t* seconds? Draw a graph that shows how the height *h* of the red tooth is determined by the elapsed time *t*. It is customary to say that *h* is a function of *t*.



2. (Continuation) Now explore the position of the red saw tooth in reference to an imagined vertical axis of symmetry of the circular blade. The red tooth is initially one foot to the right of the dotted line. How far to the right of this axis is the tooth after 37 seconds? After 237 seconds? After *t* seconds? Draw a graph that shows how the displacement *p* of the red tooth with respect to the vertical axis is a function of the elapsed time *t*.



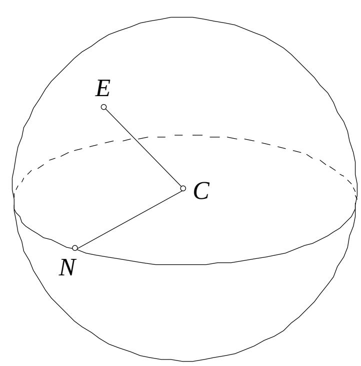
3. (Continuation) The graphs of the height *h* and the horizontal displacement *p* of the red saw tooth are examples of *sine* and *cosine* curves, respectively. Graph the equations $y = \sin x$ and $y = \cos x$ on your calculator, and compare these graphs with the graphs that you drew in the preceding exercises. Use the calculator graphs to answer the following questions:
   (a) For what values of *t* is the red tooth 0.8 feet above the table? 0.8 feet below the table?
   (b) When is the tooth 6 inches to the right of the vertical axis? When is it furthest to the left?

4. When asked to simplify the expression $\sin(180 - q)$, Pat offered the following solution: $\sin(180 - q) = \sin 180 - \sin q$, and because $\sin 180$ is zero, it follows that $\sin(180 - q)$ is the same as $-\sin q$. Is this answer correct? If not, what is a correct way to express $\sin(180 - q)$ in simpler form? Answer the same question for $\cos(180 - q)$.

5. Find simpler, equivalent expressions for the following:
   (a) $\sin(180 + q)$ (b) $\cos(180 + q)$ (c) $\tan(180 + q)$

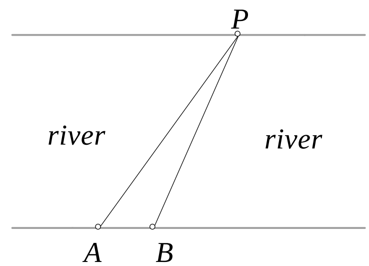
6. Show that there are at least two ways to calculate the angle formed by the vectors $[\cos 19, \sin 19]$ and $[\cos 54, \sin 54]$.

7. Let *N* be the point on the equator that is closest to *E* = Exeter, and let *C* be the center of the Earth. The central angle *ECN* is called the *latitude* of *E*; it is approximately 43 degrees. Take the radius of the Earth to be 3960 miles as you answer the following distance questions:
   (a) How far from the equator is Exeter? Travel *on* the Earth, not through it.
   (b) How far does the Earth's rotation carry the citizens of Exeter during a single day?



1. Recall that the transformation represented by the coefficient matrix $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ could also be described by the rule $\mathsf{T}(x,y) = (3x-4y, 4x+3y)$.
(a) Apply transformation T to the *unit square* whose vertices are (0,0), (1,0), (1,1), and (0,1). In particular, notice what the images of the points (1,0) and (0,1) are, and compare them with the entries in the columns of the coefficient matrix.
(b) Confirm that the same results can be obtained by doing some matrix arithmetic: Calculate the products $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and interpret the results.

2. Two industrious PEA students are trying to find the distance across the Squamscot River. After marking points *A* and *B* sixty meters apart on one bank, they sight the Powderhouse *P* on the opposite bank, and measure angles *PAB* and *PBA* to be 54 and 114 degrees, respectively. This enables them to calculate the altitude from *P* to the baseline *AB*. To the nearest meter, what was their result?



3. If sin *A* is known to be 0.96, then what can be said about cos *A*? What if it is also known that *A* is an obtuse angle?

4. The vector that results from projecting [1,1,1] onto [0,–1,0] is called a *vector projection*. Find its components.

5. Devon's bicycle, whose wheels have radius *r* meters, picks up a tack in the front wheel. After rolling another *d* meters, Devon stops the bicycle. How far above the ground is the tack?

6. (Continuation) Through how many degrees does the wheel turn for each meter that it rolls?

7. Two fire wardens are stationed at locations *P* and *Q*, which are 45.0 km apart. A forest fire *F* is sighted by each of the wardens. Given that angle *FPQ* is 52.0 degrees and angle *FQP* is 43.0 degrees, find the distance from *F* to the nearer warden, to the nearest tenth of a km.

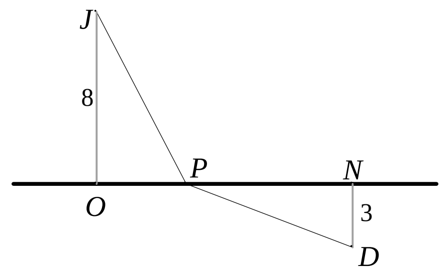
8. Find the area of a triangle that has a 10-inch side, a 17-inch side, and a 21-inch side.

9. For each of the following coefficient matrices, describe the effect of its transformation:
(a) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 5/13 & -12/13 \\ 12/13 & 5/13 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

10. (Continuation) Which of the preceding matrices represent isometries? In order for a matrix to represent an isometry, what must be true of its column vectors?

11. Explain why $|\mathbf{u}+\mathbf{v}| \le |\mathbf{u}| + |\mathbf{v}|$ holds for any vectors **u** and **v**. This is *the triangle inequality*.

1. Jamie is at the point $J = (0,8)$ offshore, needing to reach the destination $D = (12,-3)$ on land as quickly as possible. The lake shore is the x-axis. Jamie is in a boat that moves at 10 uph, with a motor bike on board that will move 20 uph once the boat reaches land. Your job is to find the landing point $P = (x,0)$ that minimizes the total travel time from J to D. Assume that the trip from P to D is along a straight line.



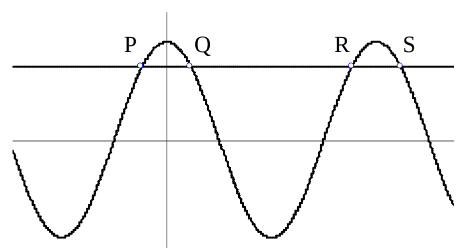
2. (Continuation) Let $O = (0,0)$ and $N = (12,0)$. Calculate the sine of angle PJO and the sine of angle PDN. These two sine values, together with the two given speeds, fit a simple relationship known as *Snell's Law*, or the *Law of Refraction*. Try to predict what you would find if the boat's speed were increased to 15 uph. To validate your prediction, you will need to solve the problem with the new speed, of course. Write a general statement of this principle.

3. Apply the 57-degree counterclockwise rotation about the origin to the vectors [1,0] and [0,1], then use the image vectors (*written as columns*) to form the coefficient matrix **M** for the rotation. Test **M** by calculating the products $\mathbf{M}\begin{bmatrix}1\\0\end{bmatrix}$ and $\mathbf{M}\begin{bmatrix}0\\1\end{bmatrix}$. Where does this rotation send the vector [3,1]? Does **M**, when applied to $\begin{bmatrix}3\\1\end{bmatrix}$, do its job correctly?

4. Write the coefficient matrix for a q-degree counterclockwise rotation about the origin.

5. Let $A = (0,0,0)$, $B = (9,8,12)$, and $C = (6,2,3)$. Find coordinates for the point on line AB that is closest to C.

6. At right you see the graphs of $y = \cos x$ and $y = 0.7431$. Given that $Q = (42, 0.7431)$, find coordinates for the intersection points P, R, and S without using a calculator. Use a calculator to check your answers.

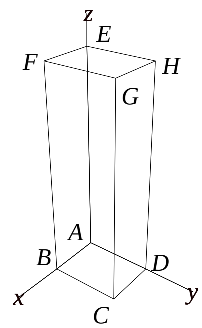


7. Verify that the circles $x^2 + y^2 = 25$ and $(x-5)^2 + (y-10)^2 = 50$ intersect at $A = (4,3)$. Find the size of the acute angle formed at A by the intersecting circles. You will first have to decide what is meant by the phrase *the angle formed by intersecting circles*.

8. (Continuation) The circles intersect at a second point B. Find coordinates for B. What can be said about the angle of intersection formed by the circles at B?

9. Let $A = (-7,-4)$ and $B = (7,4)$, and consider the equation $\overrightarrow{PA} \bullet \overrightarrow{PB} = 0$. Describe the configuration of all points $P = (x, y)$ that solve this equation.

1. The diagram at right shows a rectangular solid, two of whose vertices are $A = (0,0,0)$ and $G = (3,4,12)$. Find angle FBH and the vector projection of $\overrightarrow{BF}$ onto $\overrightarrow{BH}$.

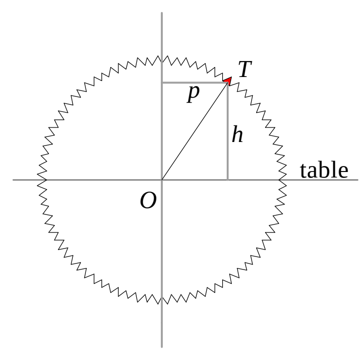


2. A paper cone has an e-inch slant height and an r-inch base radius. In terms of the quantities e and r, write a formula for the lateral surface area of the cone. In other words, find the area of the circular sector obtained by cutting the cone from base to vertex and flattening it out.

3. To the nearest tenth of a degree, find at least three solutions to each of the following:
(a) $\sin A = 0.80902$ (b) $\cos B = -0.80902$ (c) $\tan C = 1.96261$

4. Let vectors **u** and **v** form an angle q when placed tail-to-tail, and let **w** be the vector projection of **v** onto **u**.
(a) Assume that q is acute and find the length p of the vector **w**. Notice that **w** points in the same direction that **u** does. Show that $\mathbf{w} = \dfrac{\mathbf{u}\bullet\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}|\mathbf{v}|\dfrac{1}{|\mathbf{u}|}\mathbf{u}$, which simplifies to just $\mathbf{w} = \dfrac{\mathbf{u}\bullet\mathbf{v}}{\mathbf{u}\bullet\mathbf{u}}\mathbf{u}$.
(b) Now assume that q is obtuse. Do **w** and **u** still point in the same direction? Does the formula in part (a) still work?

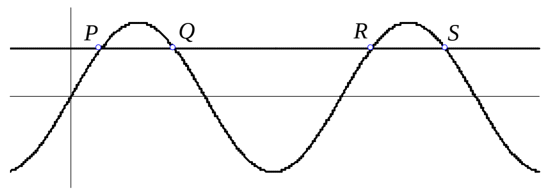
5. Revisit the circular saw blade with the one red tooth. Look at the ratio m of the height h to the horizontal displacement p. (So $m = h/p$.) Recall that the red tooth starts at the rightmost point of the saw and rotates at one degree per second. What is m after 37 seconds? After 137 seconds? After 237 seconds? After t seconds? Draw a graph that shows how m is a function of the elapsed time t. What does the ratio $m = h/p$ tell you about the line OT from the saw center to the tooth?



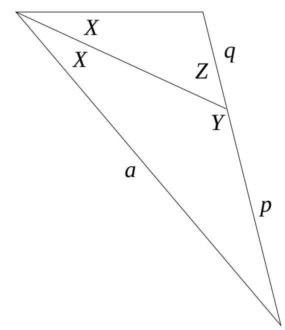
6. (Continuation) The graph of m versus t is an example of a tangent curve. Graph $y = \tan x$ on your calculator and compare it to the graph you drew in the previous exercise. Use the calculator graph to determine the values of t for which m takes on the following values: 0, 0.5, and -2. How large can m be? Is m defined for *all* values of t?

7. Explain the significance of the entries in each column of the following:
(a) the 2-by-2 matrix **M** for the reflection across the line $y = x$.
(b) the 2-by-2 matrix **N** for the 90-degree counterclockwise rotation about the origin.
(c) the product **MN**; what transformation does this represent?
(d) the product **NM**; what transformation does this represent?
(e) the product **MM**; what transformation does this represent?

8. The graphs of $y = \sin x$ and $y = k$ are shown at right. Given that the coordinates of P are (q, k), find the coordinates of Q, R, and S, in terms of q and k.



1. Refer to the diagram at right for the following questions: Express the ratio *p:a* in terms of sin *X* and sin *Y*. Express the ratio *q:c* in terms of sin *X* and sin *Z*. Because angles *Y* and *Z* are supplementary, you can now combine the preceding answers to obtain a familiar result about angle bisectors.



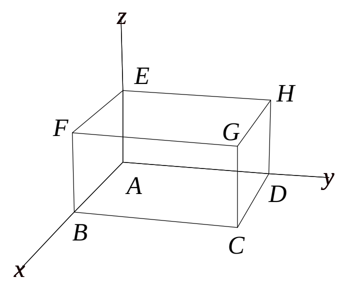
2. An amateur fruit slicer misses the center of a spherical grapefruit by one inch. The diameter of the grapefruit is 6.0 inches. What is the radius of the circular slice?

3. The point P = (–5,8) is in the second quadrant. You are used to describing it by using the *rectangular coordinates* –5 and 8. It is also possible to accurately describe the location of P by using a different pair of coordinates: its *distance from the origin* and an *angle in standard position*. These numbers are called *polar coordinates*. Calculate polar coordinates for P, and notice that there is more than one correct answer.

4. The matrix product $\begin{pmatrix} \cos 32 & -\sin 32 \\ \sin 32 & \cos 32 \end{pmatrix}\begin{pmatrix} \cos 40 & -\sin 40 \\ \sin 40 & \cos 40 \end{pmatrix}$ is equivalent to $\begin{pmatrix} \cos 72 & -\sin 72 \\ \sin 72 & \cos 72 \end{pmatrix}$. Verify this statement, then explain why this result could have been expected.

5. A sphere of radius *r* inches is sliced by a plane that is *d* inches from the center. In terms of *r* and *d*, what is the radius of the circle of intersection?

6. The diagram at right shows a rectangular solid, two of whose vertices are A = (0,0,0) and G = (4,6,3).
   (a) Find angle *FCH*.
   (b) Find vector projections of $\overrightarrow{AG}$ onto the following vectors: $\overrightarrow{AB}$, [0,1,0], [–1,0,0], and [0,0,1].
   (c) Find the point on *AC* that is closest to the midpoint of *GH*.



7. The result of reflecting across the line $y = -x$ and then rotating 330 degrees counterclockwise around the origin is an isometry T. Represent T by a 2-by-2 matrix. There is more than one way to do it. Use the point (1,1) to check your answer.

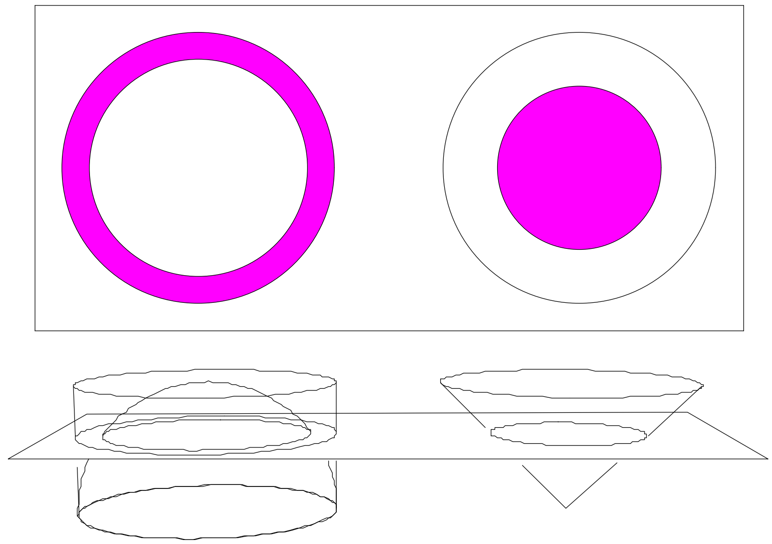
8. Given points *A* and *B* in 3-dimensional space, describe all the solutions to $\overrightarrow{PA} \bullet \overrightarrow{PB} = 0$.

9. To win the carnival game *Ring Ding*, you must toss a wooden ring onto a grid of rectangles so that it lands without touching any of the grid lines. The ring has a 3-inch diameter, the rectangles are twice as long as they are wide, and the game has been designed so that you have a 28% chance of winning. What are the dimensions of each rectangle?

10. Simplify the following:
    (a) $\cos(360-q)$ (b) $\sin(360-q)$ (c) $\cos(360+q)$ (d) $\sin(360+q)$ (e) $\tan(360+q)$

1. A hemisphere of radius r is inscribed in a cylinder of radius r and height r. The figure shows side and top views of the hemisphere, the cylinder, and a cone whose radius and height are both r, and whose base and vertex are coplanar with the bases of the cylinder.

   Consider that part of the cylinder that is outside (above) the hemisphere. When this region is sliced by a plane that is parallel to the cylinder base, the intersection is a *ring* between two concentric circles. The same plane slices the cone, creating a circular disk. Show that *the ring and the disk have the same area, no matter what the height of the slicing plane*.



2. (Continuation) If the cone were filled with liquid, it could be poured into the cylinder, which still has the hemisphere stuck in the bottom. Will all the liquid fit? Expressed in terms of r, what is the volume of the cone? of the empty cylinder? of the hemisphere?

3. (Continuation) Show that a sphere of radius r encloses a volume of $\frac{4}{3}\pi r^3$.

4. Calculate the matrix product $\begin{bmatrix} \cos 47 & -\sin 47 \\ \sin 47 & \cos 47 \end{bmatrix}\begin{bmatrix} \cos 47 & \sin 47 \\ -\sin 47 & \cos 47 \end{bmatrix}$. Interpret the result.

5. A parallelogram has a 5-inch side and an 8-inch side that make a 50-degree angle. Find the area of the parallelogram and the lengths of its diagonals.

6. Let $\mathbf{M}$ be the matrix $\begin{bmatrix} 13 & 7 \\ 5 & 3 \end{bmatrix}$. Use your calculator to evaluate $\mathbf{MM}^{-1}$. The matrix $\mathbf{M}^{-1}$ is called the *inverse* of matrix $\mathbf{M}$. Repeat this question, for another square $\mathbf{M}$ of your choosing.

7. Find the diameter of the circle that can be circumscribed around a triangle that has two 13-inch sides and one 10-inch side.

1. What does the SAS version of the Law of Cosines have to say about the “triangle” whose sides p and q form a 180-degree angle?

2. For each of the following, calculate **MN** and **NM**:

   (a) $\mathbf{M} = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ -\frac{3}{11} & \frac{5}{11} \end{bmatrix}$ (b) $\mathbf{M} = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} 0 & -1 & -1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -1 & -2 \end{bmatrix}$

   (c) $\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$

3. (Continuation) The matrix that results each time that **M** is multiplied by **N** is called the *identity matrix* **I**, and **N** is usually written as $\mathbf{M}^{-1}$. Without using your calculator, try to find $\mathbf{M}^{-1}$ for the matrix $\mathbf{M} = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$.

4. Polar coordinates for a point P in the xy-plane consist of two numbers r and q, where r is the distance from P to the origin O, and q is the size of an angle in standard position that has OP as its terminal ray. Find polar coordinates for each of the following points:
   (a) (0,1) (b) (–1,1) (c) (4,–3) (d) (1,7) (e) (–1,–7)

5. Triangle KLM has a 120-degree angle at K and side KL is three fourths as long as side LM. To the nearest tenth of a degree, find the sizes of the other two angles of this triangle.

6. A drinking cup is 27/64 full of liquid. What is the ratio of the depth of the liquid to the depth of the cup, assuming that (a) the cup is cylindrical? (b) the cup is conical?

7. A jet leaves Oslo, whose latitude is 60 degrees north of the equator, and flies due west until it returns to Oslo. How far does the jet travel? The radius of the Earth is 3960 miles.

8. The rectangle shown has been formed by fitting together four right triangles. As marked, the sizes of two of the angles are a and b (*Greek* “alpha” and “beta”), and the length of one segment is 1. Find the two unmarked angles whose sizes are a and $a+b$. By labeling all the segments of the diagram, discover formulas for $\sin(a+b)$ and $\cos(a+b)$, written in terms of $\sin a$, $\cos a$, $\sin b$, and $\cos b$.