

(a) Evaluate the inverse Laplace transform of the expression

$$\frac{s^3 + s^2 + 2s - 1}{s^2(s^2 + 3s + 2)}$$

Hence, or otherwise, solve the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 1 - t \quad y(0) = 1, \quad y'(0) = -2.$$

(b) Sketch the graphs and obtain the Laplace transforms of the functions defined by

(i) $f(t) = (1 + t^2)[u(t - 1) - u(t - 2)]$ where $u(t)$ is the unit step function.

(ii) $f(t) = \begin{cases} \sin t & 0 \leq t \leq \pi \\ 0 & \pi \leq t \leq 2\pi \end{cases} \quad f(t + 2\pi) = f(t)$