

(16) Let $P_2(x)$ be the quadratic polynomial interpolating $f(x)$ at the evenly spaced points $x_0, x_1 = x_0 + h, x_2 = x_0 + 2h$. Derive formulas for the error:

$$f'(x_i) - P_2'(x_i), \quad i = 0, 1, 2$$

Assuming $f(x)$ is three times continuously differentiable, give computable bounds for these errors.

Hint: use the error formula:

$$f(t) - P_n(t) = (t - x_0) \cdots (t - x_n) f[x_0, \dots, x_n, t]$$