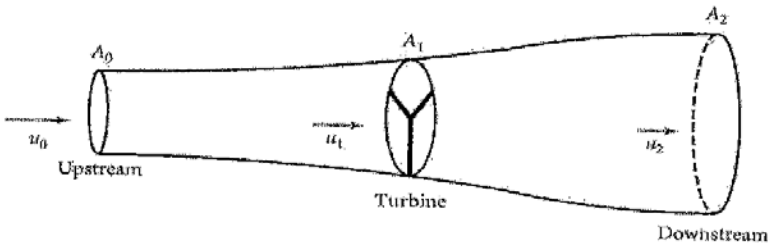
2 (b) The diagram below shows a stream-tube in which the rate of flow of mass is dm/dt.

Write down an expression for dm/dt at the turbine in terms of the density of air ρ.



The power P generated is equal to the rate of change of kinetic energy

$$P = \tfrac{1}{2}\, dm/dt\, (u_0^2 - u_2^2) \quad (i).$$

Show that an alternative expression is given by

$$P = dm/dt\, (u_0 - u_2)\, u_1 \quad (ii).$$

Hence show that $u_2 = 2\, u_1 - u_0$.

Substituting in (ii) and writing $u_1 = u_0\,(1\text{-}a)$ show that

$$P = \tfrac{1}{2}\, \rho\, u_0^3 A_1 C_P \text{ where the power coefficient } C_P = 4a(1\text{-}a)^2.$$

Show that the Betz limit for the power coefficient is $C_P = 16 / 27$.