#13. We say an $n \times n$ matrix A is *orthogonal* if $A^{\mathsf{T}}A = I_n$.

a. Prove that the column vectors $\mathbf{a}_1, \ldots, \mathbf{a}_n$ of an orthogonal matrix A are unit vectors that are orthogonal to one another, i.e.,

$$\mathbf{a}_i \cdot \mathbf{a}_j = \begin{cases} 1, & i = j \\ 0, & i \neq j. \end{cases}$$

b. Fill in the missing columns in the following matrices to make them orthogonal.

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & ? \\ -\frac{1}{2} & ? \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & ? \\ 0 & -1 & ? \\ 0 & 0 & ? \end{bmatrix}, \quad \begin{bmatrix} \frac{1}{3} & ? & \frac{2}{3} \\ \frac{2}{3} & ? & -\frac{2}{3} \\ \frac{2}{3} & ? & \frac{1}{3} \end{bmatrix}$$