6. A particle of mass m is in the ground state of a potential described by $V(x) = \frac{1}{2}m\omega^2 x^2$ when the equilibrium point of the potential is suddenly changed from $x = 0$ to $x = s$. Show that the probability that after this change the particle will be in the first excited state of the new potential is given by

$$P_1 = (\alpha s)^2 \exp\left(-\frac{(\alpha s)^2}{2}\right).$$

Find the value of s which maximizes this probability. Explain whether the value of s which maximizes the probability of excitation to a high-lying energy level would be bigger or smaller than this.

[With $\alpha = \sqrt{m\omega/\hbar}$, the normalized wave functions of the particle's two lowest states are

$$\phi_0(x) = \frac{\alpha^{1/2}}{\pi^{1/4}}\,\mathrm{e}^{-\alpha^2 x^2/2} \quad \text{and} \quad \phi_1(x) = \frac{2\alpha^{3/2}}{\pi^{1/4}}\,x\,\mathrm{e}^{-\alpha^2 x^2/2}.$$

The following integral may be useful:

$$\int_{-\infty}^{\infty} (x-b)\exp\left\{-a^2\left(x-\frac{b}{2}\right)^2\right\}\,\mathrm{d}x = -\frac{b\sqrt{\pi}}{2a} \quad \text{if } a, b > 0.\Big]$$