Consider an electromagnetic wave travelling through a homogeneous, isotropic medium of conductivity σ, relative permittivity ε_r and relative permeability μ_r Use Maxwell's equations to show that the wave equation for the electric field **E** is

$$\nabla^2\mathbf{E} - \sigma\mu_0\mu_r\frac{d\mathbf{E}}{dt} - \varepsilon_0\mu_0\varepsilon_r\mu_r\frac{d^2\mathbf{E}}{dt^2} = 0$$

Show that for a plane wave, solutions to the wave equation satisfy the relation

$$k^2 = \varepsilon_0\mu_0\varepsilon_r\mu_r\omega^2\left(1 + \frac{i\,\sigma}{\varepsilon_r\varepsilon_0\omega}\right)$$

where the wave has wavenumber k and angular frequency ω. Show that the wave amplitude is attenuated exponentially as it propagates through the conducting medium, and derive an expression for the attenuation length, also known as the skin depth, in the limit of high conductivity.

Give an example of when such attenuation in a conducting medium is of practical importance. Calculate the skin depth in copper at a frequency of 50Hz. You may assume $\mu_r = 1$ for copper.

[Copper has conductivity $5.8*10^7$ siemens m^{-1}]