A. Basic Mathematics

1. Given that the Taylor series for the function

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...,$$

use this to show the following

$$\begin{aligned} \frac{x}{\left(1+x^2\right)^2} &= x - 2x^3 + 3x^5 - .. \\ \log\left(1-x\right) &= -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - .. \end{aligned}$$

2. Consider complex number division $\frac{a+ib}{c+id}$ which we express as $re^{i\alpha}$. Using Euler's identity **only**, work out the precise form for the quotient's modulus r and argument α. Your calculations should not involve divsion of complex numbers.

3. Using row operations evaluate the following determinant $\begin{vmatrix} y-z & z-x & x-y \\ z-x & x-y & y-z \\ x-y & y-z & z-x \end{vmatrix}$

4. Find the eigenvalues and eigenvectors of the following matrix

$$\begin{pmatrix} 3 & 3 & 3 \\ 3 & -1 & 1 \\ 3 & 1 & -1 \end{pmatrix}.$$

Verify that the eigenvectors are mutually orthogonal and hence diagonalise the matrix. Show all working.