Redshift

(a) Describe what is meant by *redshift* and how spectroscopic observations of extragalactic objects may be used to deduce their redshifts.

(b) What is meant by the term *metric*? The Friedmann–Robertson–Walker metric for a homogeneous and isotropic Universe is given by

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right],$$

where $\mathrm{d}s$ is the proper time interval between two events, t is the cosmic time, k measures the spatial curvature, r, θ and ϕ are radial, polar and azimuthal co-ordinates respectively. Explain what is meant by $a(t)$ and discuss its physical significance.

(c) What does the above expression become in the case of a light-ray? Hence derive an integral expression for a light-ray which leaves the origin at time t_{em} and reaches a co-moving distance r_0 at time t_{obs}. A second ray is emitted a time $\mathrm{d}t$ after the first. By considering the two intervals as corresponding to successive wave crests, derive the relation

$$\frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{em}}} = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{em}})} \equiv 1 + z,$$

where z is the redshift and λ_{em} and λ_{obs} are the emitted and observed wavelengths respectively.

(d) How does the separation of galaxies today compare with the separation of galaxies when light left the galaxies we observe at redshift 1?