1. Evaluate the following indefinite integrals:

1. $\int \frac{1}{x^2} \mathrm{d}x$
2. $\int e^{-x} \mathrm{d}x$
3. $\int \sin(t)\cos(t)\mathrm{d}t$
4. $\int \sqrt{s}\ \mathrm{d}s$

2. On a dark night in 1915, a German zeppelin bomber drifts menacingly over London. The men on the ground train a spotlight on the airship, which is traveling at 90 km/hour, and at a constant altitude of 1 km. The beam of the spotlight makes an angle θ with the ground.

1. Draw a diagram of the situation.
2. When the airship is 3 kilometers from the spotlight, how fast is θ changing?

3. An electrical circuit consists of two parallel resistors, with resistances R_1 and R_2 respectively. The total resistance R of the circuit (measured in Ohms Ω) is specified by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

The resistors are heating up, so their resistances are increasing over time. Suppose that R_1 is increasing at a rate of $0.3\Omega/s$ and R_2 is increasing at a rate of $0.2\Omega/s$. When $R_1 = 80\Omega$ and $R_2 = 100\Omega$, how fast is the total resistance increasing?

4. The ideal gas law states that the pressure P, volume V, and temperature T of a gas are related by

$$PV = NkT$$

where N is the number of molecules of gas, and k is Bolzmann's constant, about $1.38 * 10^{-23} J/K$ where J is Joules and K is Kelvins (look it up on Wikipedia if you're curious). Say that I have 10^{24} molecules of gas. The gas begins at a pressure of 200 kPa, inside a 100 cm^3 container, and at a temperature of $400K$.

1. Say that I hold the temperature fixed at $400K$ and begin to decrease the volume of the container at a rate of 10 cm^3/s. At what rate is the pressure changing?

2. What if instead the pressure is kept fixed at 200 kPa, and the gas is cooled at a rate of $-2K/s$. At what rate is the volume changing?

5. Use the fundamental theorem of calculus to evaluate $\int_0^\pi \sin(x)\mathrm{d}x$. You must show your work.

6. What is $\frac{d}{dx}\int_0^x \exp(\sin(s) + s^s)\mathrm{d}s$? (Hint: use the fundamental theorem of calculus.)

7. Approximate $\int_0^1 \exp(-t^2)\mathrm{d}t$ by partitioning the interval $[0, 1]$ into 5 parts of equal length and sampling $\exp(-t^2)$ at the midpoint of each part. Round to three decimal places.

8. The graph of 2^x crosses the graph of x^2 at $x = 2$ and $x = 4$; see the diagram below. Given that $\int 2^x\mathrm{d}x = (1/\ln(2))2^x + C$, find the area of the region A enclosed by the two curves.