a) Write a state equation for this system:

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=u$$

b) Add a new state in which $\dot{x}_3=y-y_d$ where y_d is the unit step and represents the desired output of the system. Write the new state equations treating y_d as another input.

c) Now, pick a vector $k=[k_1,k_2,k_3]$ such that if $u=-kx$, the new state will have roots of the characteristic equation at -2, -3, and -4

d) Simulate the system and observe the output y and compare the response for y_d=step input to the step response of the system without control.

Helpful notes:

a) Use state variables $x_1=y$ and $x_2=\dot{x}_1$. You can then write down the state equation as: $\dot{x}=Ax+Bx$
A is a 2x2 matrix, x & $\dot{x}$ are 2x1, and B is 2x1 with (u) as the scalar input.

b) The part expresses the integrated difference of output y and desired output y_d. this is expressed as $\dot{x}_3=y-y_d$. since $x_1=y$, the equation will look like: $\dot{x}_3=x_1-y_d$. and if this is included in the state equation form problem a), we get a 3x3 matrix (A) and 3x2 matrix (B) relating derivatives to the state variables to input, u and y_d. The matrix will look like:

$$\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu}$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\end{pmatrix}\begin{pmatrix}u\\ y_d\end{pmatrix}$$

We need to find the values of the a's and b's.

c) The goal is to create a controller with a desired response by replacing the input u by $-\mathbf{kx}$ where **k** is a row vector of constants and **x** a column vector of states: $\bar{k}=(k_1 \quad k_2 \quad k_3)$ and $\bar{x}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$

We must first substitute –kx for (u) in the equation $\dot{\mathbf{x}}=\mathbf{Ax}+\mathbf{Bu}$ and we are asked to find the values of $k_{1,2\&3}$ that make the characteristic equation for this system match a desired characteristic equation with roots -2,-3, and -4.
Given the roots the desired characteristic equation, we can write the characteristic equation as: (s+2)(s+3)(s+4) and expand this formal to get s^3+es^2+fs+g where we need to find the values of *e*, *f* & *g*.