3-9 *Why can bound-state wave functions be chosen to be real?* The text states that, in one-dimensional problems, the spatial wave function for any allowed state can be chosen to be real-valued. Verify this using the following outline or some other method.

(a) Write the wave function $\psi_n(x)$ in terms of its real and imaginary parts: $\psi_n = \text{Re}\ \psi_n + i \cdot \text{Im}\ \psi_n$, and substitute this into the Schrödinger equation.

(b) Show that Re ψ_n and Im ψ_n separately satisfy the Schrödinger equation.

(c) In one dimension there is only one (linearly independent) wave function for each energy eigenvalue E_n. What does this imply about Re ψ_n and Im ψ_n? Describe how you can construct a real, normalized wave function from ψ_n.