(c) **[7 marks]** One student claims that the following integral is path-independent:

$$I = \int_C -2y\cos 2x\,dx - \sin 2x\,dy$$

Is this statement correct? Justify your answer. Find the integral in case the path is given by: $x(t) = t^3$; $y(t) = t^4$; $0 \le t \le 1$.

(b) **[6 marks]** Let $\underline{r} = (x, y, z)$ and $r = |\underline{r}|$. Show that:

(i) $\nabla(\ln r) = \frac{\underline{r}}{r^2}$;

(ii) $\nabla(r^n) = nr^{n-2}\underline{r}$.

(c) **[7 marks]** Consider the transformation from Cartesian coordinates $(x, y) > 0$ into the coordinates given by:

$$x = \sqrt{\frac{u}{v}};\ y = \sqrt{uv},$$

where $u, v > 0$. Calculate the Jacobian of this transformation.

(d) [$7\frac{1}{3}$ marks] Let $\underline{F}(x,y,z) = (x^2, y^3, z)$ vector field, and let a curve C be parametrised by

$$\underline{r}(t) = (t, |t|, 0), \quad t \in [-1, 1].$$

Find the line integral $\int_C \underline{F} \cdot \mathrm{d}\underline{s}$ of $\underline{F}$ along C.

(c) **[7 marks]** Consider the transformation from Cartesian coordinates $(x, y) > 0$ into the coordinates given by:

$$x = \sqrt{\frac{u}{v}}; \; y = \sqrt{uv},$$

where $u, v > 0$. Calculate the Jacobian of this transformation.

(d) **[7 marks]** Sketch the domain enclosed by the curve:

$$\left(x^2 + y^2\right)^2 = 2a^2 xy$$

and evaluate its area.

(a) [6 marks] Consider the transformation of coordinates from Cartesian to spherical coordinates:

$$x(r,\theta,\phi) = r\cos\theta\sin\phi, \quad y(r,\theta,\phi) = r\sin\theta\sin\phi, \quad z(r,\theta,\phi) = r\cos\phi,$$

for $r \geq 0$, $0 \leq \theta < 2\pi$, and $0 \leq \phi \leq \pi$. Describe the spherical coordinate system geometrically. Calculate the Jacobian $J(r,\theta,\phi)$ for this transformation of coordinates.

(e) **[7 marks]** Evaluate the following triple integral :

$$\int_{-1}^{1} dx \int_{x^2}^{1} dy \int_{0}^{2} (4+z)dz.$$

Construct the domain of integration.