3.1 Mixing the Constant with the Normal

Find and expression for the joint p.d.f. of a distribution for which the following is true:

(a). The conditional distribution of Y given $X = t$ is normal for $t \in (a, b)$.

(b). $E(Y|X = t)$ is a linear function in t for $t \in (a, b)$.

(c). $\sigma_{Y|X=t}$ is constant for all values of $t \in (a, b)$.

(d). The marginal distribution in X is a constant, i.e.

$$f_X(t) = \begin{cases} \frac{1}{a+b} & \text{for } t \in (a, b) \\ 0 & \text{otherwise} \end{cases}$$