1. Find $f'(z)$ when

   1) $f(z) = 3z^2 + 2iz - i$
   2) $f(z) = (1 - 4z^2)^3$
   3) $f(z) = \dfrac{(1 + iz^2)^4}{z^2}$

2. Show that $f'(z)$ does not exist at any point $z \in \mathbb{C}$ when

   1) $f(z) = \overline{z}$
   2) $f(z) = \operatorname{Re} z$

3. Let $f(x + iy) = e^x(\cos y + i \sin y)$ for all $x, y \in \mathbb{R}$. Show that f is an entire function, and find the derivative of f.

4. Let Ω be a domain in the complex plane, and let f be a function analytic on Ω. Using the fact that

$$\text{if } f'(z) = 0 \text{ for all } z \in \Omega, \text{ then } f \text{ is a constant function on } \Omega,$$

   show that

   1) f is a constant function on Ω if $\operatorname{Re} f$ is a constant function on Ω
   2) f is a constant function on Ω if $\operatorname{Im} f$ is a constant function on Ω

5. Let

$$\Omega = \{r(\cos\theta + i \sin\theta) : r > 0 \text{ and } -\pi < \theta < \pi\}.$$

   1) For $r > 0$ and $-\pi < \theta < \pi$, let

$$f(r\cos\theta + ir\sin\theta) = \sqrt{r}(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}).$$

      Show that f is analytic on Ω.

   2) For $r > 0$ and $-\pi < \theta < \pi$, let

$$g(r\cos\theta + ir\sin\theta) = \ln r + i\theta,$$

      here $\ln r$ denotes the natural logarithmic function defined for positive real number r. Show that g is analytic on Ω.