Group 6

Airlines sometimes overbook flights (that is, they sell more tickets than there are seats on the plane). Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable X as the number of ticketed passengers who actually show up for the flight. The probability distribution function of X is shown in the table below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 |
| *p(x)* | 0.05 | 0.10 | 0.12 | 0.14 | 0.25 | 0.17 | 0.06 | 0.05 | 0.03 | 0.02 | 0.01 |

**Question 1**. What is the probability that the flight will accommodate all ticketed passengers who show up?



**Question 2**. Calculate E(X), the expected number of ticketed passengers who show up for the flight.



**Question 3**. If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight?



**Question 4**. If you are the third person on the standby list, what is the probability that you will be able to take the flight?

**2.**

**2.5/2.5 points**

 |  Previous Answers

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 |
| **1/1** | **1.5/1.5** |
| 3/4 | 4/4 |
| **Total** |
| **2.5/2.5** |
|  |

You are planning to take to a trip to Montreal, Canada during the month of April and you want to bring clothing that is appropriate for the weather. The daily high temperature X in degrees Celsius in Montreal during April has expected value E(X) = 9.5oC with a standard deviation SD(X) = 3.5oC. You want to convert these Celsius temperatures to oF (degrees Fahrenheit). The conversion of X into degrees Fahrenheit Y is Y = (9/5)X + 32.

**Question 1.** What is E(Y), the expected daily high in Montreal during April in degrees Fahrenheit?



**Question 2.** What is SD(Y), the standard deviation of the daily high temperature in Montreal during April in degrees Fahrenheit? 6.19



**3.**

**1/4.5 points**

 |  Previous Answers

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 | 3 | 4 |
| **1/1** | **0/1.5** | **0/1** | **–/1** |
| 1/4 | 2/4 | 2/4 | 0/4 |
| **Total** |
| **1/4.5** |
|  |

In a process for manufacturing glassware, glass stems are sealed by heating them in a flame. The temperature of the flame varies a bit. Here is the distribution of the temperature X measured in degrees Celsius:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Temperature X** | 540o | 545o | 550o | 555o | 560o |
| **Probability** | 0.1 | 0.25 | 0.3 | 0.25 | 0.1 |

**a.** Find the expected value of the temperature X.



**b.** Find the standard deviation of the temperature X.



(Use 3 decimal places.)

The target temperature is 550oC.

**c.** What is the expected value of the number of degrees off target, X-550?



**d.** What is the standard deviation of the number of degrees off target, X-550?

**4.**

**–/5 points**

**My Notes**

"Digital Analysis" is an important new tool auditors use when looking for fraud. Faked numbers in payment records, invoices, expense account claims, and many other settings often display patterns that are NOT present in legitimate records.

Some patterns, like too many round numbers, are obvious and easily avoided by a clever crook. Others are more subtle. It is a surprising fact that the first digit X of numbers in legitimate records are NOT equally distributed between 1 and 9, but follow a distribution known as ***Benford's Law***. The distribution of the first digit according to Benford's Law is shown in the first table below.

**Benford's Law**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| First Digit X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Probability | .301 | .176 | .125 | .097 | .079 | .067 | .058 | .051 | .046 |

If first digits in a set of records appeared "at random," the nine possible digits 1 to 9 all have the same probability, that is, each of the digits 1, 2, . . . , 9 is equally likely to be the first digit. The probability distribution of the first digit Y according to the random model is shown in the table below.

**Random Model**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| First Digit Y | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Probability | 1/9 | 1/9 | 1/9 | 1/9 | 1/9 | 1/9 | 1/9 | 1/9 | 1/9 |

An auditor or fraud investigator can put a computer to work comparing invoices from your company's vendors with the distribution shown in Benford's Law. A vendor whose invoices show a different pattern can be investigated further for any "Enron-type" activities.

It should be noted that not all sets of data follow Benford's Law. Numbers that are assigned, such as social security numbers, or data with a fixed maximum, such as deductible contributions to individual retirement accounts, or randomly generated numbers, do not. But a surprising variety of data from natural science, social affairs, and business obeys Benford's Law.

**Questions:**

**(a)**. According to Benford's Law, what is the probability that a first digit is a 1 or 2? (Use 3 decimal places).

**(b)**. According to the "random" model, what is the probability that a first digit is a 1 or 2? (Use 3 decimal places).

**(c)**. According to Benford's Law, what is the probability that a first digit is greater than or equal to 6? (Use 3 decimal places).

**(d)**. According to the "random" model what is the probability that a first digit is greater than or equal to 6? (Use 3 decimal places).

Benford's Law allows easy detection of phony financial records based on randomly generated numbers; records of the latter type tend to have too few first digits being 1s and 2s, and too many first digits of 6 or greater.

**(e)**. What is the expected value of the first digit X when the first digit follows Benford's Law? E(X) = (Use 3 decimal places).

**(f)**. What is the expected value of the first digit Y when the first digit appears at random? E(Y) =

**(g)**. What is the standard deviation of the first digit X when the first digit follows Benford's Law? SD(X) = (Use 3 decimal places).

**5.**

**–/3 points**

**My Notes**

Investors, be they large corporations, banks, pension funds, mutual funds, or individuals, seldom hold a single financial asset; rather, they hold portfolios of financial assets. Thus, the investor should be less concerned with the rate of return achieved by, say a particular stock in the portfolio than the overall rate of return of the portfolio.

Typically, an investor can choose from among many different assets to form a portfolio; in other words, there are many different portfolios from which an investor can choose. But which portfolio should the investor select? Since the future rate of return of a portfolio is uncertain a *probability distribution* can be used to characterize a portfolio's future rate of return. In particular, **investment managers frequently characterize portfolios by the mean mu (expected value) and standard deviation sigma (risk) of their rates of return**. First, the set of all possible portfolios - - the ***feasible set*** - - must be reduced to an ***efficient set*** of portfolios. An efficient portfolio is one that provides the highest possible mean rate of return for any given degree of risk (i.e., any given standard deviation sigma) or the lowest possible degree of risk (i.e., the lowest standard deviation sigma) for any given mean rate of return.

The attached graph [(right click here and choose "Open Link in New Window" to see graph)](http://www.stat.ncsu.edu/people/reiland/courses/st350/hw9port.htm) shows the mean and standard deviation of the rate of return for various portfolios. Portfolios identified by a value of mu and sigma that fall within the ellipse represent the *feasible set* of portfolios for a particular investor. The *efficient set* of portfolios is denoted by the boundary arc *ABC* and is sometimes called the *efficient frontier*. Portfolios to the left of *ABC* are not attainable because they fall outside the feasible set. Portfolios to the right of *ABC* are not efficient because there always exists a portfolio that could provide (1) a higher mean return for a given level of standard deviation of returns (compare points *B* and *D*), or (2) a lower standard deviation (lower degree of risk) for a given mean rate of return (compare points *B* and *E* which have the same mean rate of return).

The future rates of return of stock portfolios *X*, *Y*, and *Z* can be characterized by the following probability distributions:

|  |
| --- |
| **STOCK PORTFOLIO X** |
| Rate of Return | Probability |
| .20 | .05 |
| .15 | .15 |
| .10 | .26 |
| .05 | .20 |
| .00 | .15 |
| -.05 | .10 |
| -.10 | .05 |
| -.15 | .03 |
| -.20 | .01 |
| **STOCK PORTFOLIO Y** |
| Rate of Return | Probability |
| .15 | .10 |
| .10 | .20 |
| .05 | .30 |
| .00 | .20 |
| -.05 | .10 |
| -.10 | .07 |
| -.15 | .03 |
| **STOCK PORTFOLIO Z** |
| Rate of Return | Probability |
| .25 | .05 |
| .20 | .10 |
| .15 | .25 |
| .10 | .20 |
| .05 | .15 |
| .00 | .10 |
| -.05 | .07 |
| -.10 | .05 |
| -.15 | .03 |

**a.** What is the probability of a negative rate of return if you invest in portfolio Y?

**b.** Find the expected rate of return of portfolio X. .

**c.** Find the expected rate of return of portfolio Y. .

**d.** Find the expected rate of return of portfolio Z. .

Suppose you plot the mean and standard deviation for each of the portfolios X, Y, and Z on a graph like the one in the above link. Note that the standard deviation of the rate of return is on the horizontal axis and the expected rate of return is on the vertical axis. (DO NOT CALCULATE THE STANDARD DEVIATIONS! The standard deviations are as follows: portfolio X: .087155; portfolio Y: .074181; portfolio Z: .0966).

**e.** Below are statements about portfolios X, Y, and Z. Choose all the statements that are true. (This is a multiple-select problem; you must choose **all** the true statements to get the problem correct).

Portfolio Y is for timid investors who want the least amount of risk.

The overall pattern in the plot of the three portfolios is from lower left to upper right, meaning that portfolios with low risk have a high expected return.

If you want to maximize your expected rate of return and are not concerned about risk, you would choose portfolio Z.

Portfolio X has more risk than portfolio Y.

The overall pattern in the plot of the three portfolios is from lower left to upper right, meaning that if you want a higher return you have to take more risk.

A young investor who wants more return and who is not very concerned about risk should choose portfolio Y over portfolio X.

An investor close to retirement who does not want to put her retirement nest egg at risk should invest in portfolio Z.

**6.**

**0.5/1.5 points**

 |  Previous Answers

**My Notes**

In the 4x100 medley relay event in swimming, four swimmers swim 100 yards, each using a different stroke. The four medley relay swimmers for a college team preparing for their conference championships have the following summary statistics (in seconds) for the four strokes:

|  |  |  |
| --- | --- | --- |
| **Swimmer** | **Mean** | **St. Dev.** |
| **1** (backstroke) | 50.72 | 0.21 |
| **2** (breaststroke) | 55.51 | 0.23 |
| **3** (butterfly) | 49.43 | 0.22 |
| **4** (freestyle) | 44.91 | 0.2 |

**Question.** Let the random variable T denote the relay team's total time in the medley event. Determine the mean E(T) and standard deviation SD(T).



mean



standard deviation (use 3 decimal places)

**7.**

**1/4 points**

 |  Previous Answers

**My Notes**

A "buy stock option" is a privilege sold by one party to another that gives the purchaser the right, but not the obligation, to buy a particular stock at an agreed-upon price on a specific date.

You have an opportunity to purchase a "buy stock option" for $200. If the stock closes above $30 on Dec. 15, the option will be worth $1000. If the stock closes below $20 on Dec. 15, the option will be worth nothing. If it closes between $20 and $30 inclusive, on Dec. 15, the option will be worth $200.

Your financial advisor tells you that there is a 50% chance that the stock will close in the $20 - $30 range, a 20% chance that it will close above $30, and a 30% chance that the stock will close below $20 on Dec. 15.

**Question 1**. Which table below shows the probability distribution for the value of the stock option on Dec. 15.

|  |  |  |  |
| --- | --- | --- | --- |
| **Stock Option Value *x*** | $0 | $200 | $1000 |
| ***p(x)*** | .50 | .30 | .20 |
| **Stock Option Value *x*** | $0 | $200 | $1000 |
| ***p(x)*** | .30 | .30 | .20 |

|  |  |  |  |
| --- | --- | --- | --- |
| **Stock Option Value *x*** | under $20 | $20 to $30, inclusive | over $30 |
| ***p(x)*** | .30 | .20 | .50 |
| **Stock Option Value *x*** | under $20 | $20 to $30, inclusive | over $30 |
| ***p(x)*** | .30 | .50 | .20 |
| **Stock Option Value *x*** | $0 | $200 | $1000 |
| ***p(x)*** | .30 | .50 | .20 |



**Question 2**. How much do you expect the stock option to be worth? (DO NOT use a dollar sign in your answer).

**Question 3**. What is the standard deviation of the stock option's worth? (DO NOT use a dollar sign in your answer).

**Question 4**. How much do you expect to gain? (DO NOT use a dollar sign in your answer).

**Question 5**. What is the standard deviation of your gain? (DO NOT use a dollar sign in your answer). .

**8.**

**–/3 points**

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 | 3 | 4 |
| **–/0.5** | **–/1** | **–/0.5** | **–/1** |
| 0/4 | 0/4 | 0/4 | 0/4 |
| **Total** |
| **–/3** |
|  |

To compete with Netflix, the owner of a local movie rental store decides to try sending DVDs through the mail. To plan for this endeavor, she sends DVDs to her friends to obtain data on delivery times when DVDs are mailed to customers and return times when DVDs are mailed back to the store.

Let the random variable X denote the delivery time when a DVD is mailed to a customer, and let the random variable Y denote the return time when a customer mails the DVD back to the store.

The store owner found that the mean delivery time, E(X), was 1.7 days, and that the standard deviation of delivery time, SD(X), is 0.6 days. She found that the mean return time, E(Y), is 2.3 days, with a standard deviation SD(Y) of 0.9 days.

**Question 1.** Determine the mean and standard deviation of the total transit time for a DVD (that is, the delivery time when mailed to the customer plus the return time when mailed back to the store). days, mean total transit time days, standard deviation of total transit time (use 3 decimal places)

**Question 2.** On average the return time Y is greater than the delivery time X. Determine the mean and standard deviation of how much the return time Y exceeds the delivery time X. days, mean of how much the return time exceeds the delivery time. days, standard deviation of how much the return time exceeds the delivery time.

**9.**

**–/3 points**

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 | 3 | 4 |
| **–/0.5** | **–/1** | **–/0.5** | **–/1** |
| 0/4 | 0/4 | 0/4 | 0/4 |
| **Total** |
| **–/3** |
|  |

A life insurance company sells a term insurance policy to a 21-year-old male that pays $100,000 if the insured dies within the next 5 years. The company collects a premium of $250 each year as payment for the insurance. The amount *X* that the company earns on this policy is $250 per year, less the $100,000 that it must pay if the insured dies. From mortality tables, the expected value of X, denoted E(X) or *μ*, is $303 and the standard deviation of X, denoted SD(X) or *σ*, is $9707.

**Question 1**. The risk of insuring one person's life is reduced if we insure many people. Suppose an insurance company insures two 21-year-old males and that their ages at death are independent. If *X* and *Y* are what the company earns from the two insurance policies, the insurance company's average income on the two policies is .

Find the expected value E(Z) and the standard deviation SD(Z) of the random variable Z. (**Note:** since the ages at death are independent, Var(X+Y)=Var(X)+Var(Y)) E(Z) = $  SD(Z) = $ .

**Question 2**. If four 21-year-old males are insured, the insurance company's average income is  where *Xi* is what the insurance company earns by insuring one man. The *Xi* are independent and each has the same distribution with expected value E(X) and standard deviation SD(X) as given above.

Find the expected value E(Z) and the standard deviation SD(Z) of the random variable Z. (**Note:** since the Xi's are independent, the variance of the sum of the Xi's is the sum of the individual variances). E(Z) = $  SD(Z) = $ .

Group 7

**1.**

**0.5/5 points**

 |  Previous Answers

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| **0/0.5** | **–/1** | **0.5/0.5** | **–/0.5** | **–/1** | **0/0.5** | **0/1** |
| 4/4 | 0/4 | 2/4 | 0/4 | 0/4 | 2/4 | 1/4 |
| **Total** |
| **0.5/5** |
|  |

Assume that 13% of people are left-handed. If we select 10 people at random, answer the following questions:

**a.** What is the probability that there are exactly 3 lefties in the group?.002



(Use 3 decimal places.)

**b.** What is the probability that there are at least 3 lefties in the group? (Use 3 decimal places.)

**c.** Find the expected number of right-handers in the group.



**d.** Find the standard deviation of the number of right-handers in the group. (Use 3 decimal places.)

**e.** What is the probability that they are not all right-handed? (Use 3 decimal places.)

**f.** What is the probability that there are exactly 5 righties and 5 lefties?



(Use 4 decimal places.)

**g.** What is the probability that the majority is right-handed?



(Use 3 decimal places.)

**2.**

**0/1 points**

 |  Previous Answers

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 |
| **0/0.5** | **0/0.5** |
| 2/4 | 1/4 |
| **Total** |
| **0/1** |
|  |

A state end-of-grade exam in American History is a multiple-choice test that has 50 questions with 4 answer choices for each question. A student must get at least 25 correct to pass the test, and the questions are very difficult.

**Question 1**. If a student guesses on every question, what is the probability the student will pass?



(round your answer to 4 decimal places).

**Question 2**. Suppose, after studying, a student raises her chances of getting each question correct to 0.70. What is the probability that she will pass?



(round your answer to 4 decimal places).

**3.**

**–/5.5 points**

**My Notes**

|  |
| --- |
| Question Part |
| Points |
| Submissions Used |
| 1 | 2 | 3 | 4 |
| **–/1** | **–/1.5** | **–/1.5** | **–/1.5** |
| 0/4 | 0/4 | 0/4 | 0/4 |
| **Total** |
| **–/5.5** |
|  |

The rules of Major League Baseball's World Series stipulate that the first team to win 4 games wins the World Series. Team A and team B are playing in the World Series and they are evenly matched (that is, each team has a .50 chance of winning each game).

You are the director of marketing for the television network that has paid millions of dollars for the rights to televise this World Series. The network CEO is concerned that the network will lose money because of insufficient advertising revenue from the games. The CEO wants you to immediately answer the following questions:

**a.** What is the probability that the World Series ends in exactly 4 games? **Hint:** There are 2 teams. (Use 3 decimal places.)

**b.** What is the probability that the World Series ends in exactly 5 games? **Hint:** Don't use 5C4. (Use 2 decimal places.)

**c.** What is the probability that the World Series ends in exactly 6 games? (Use 4 decimal places.)

**d.** What is the probability that the World Series ends in exactly 7 games? (Use 4 decimal places.)

**Note:** The players on the participating teams in the World Series receive money generated only by the first 4 games. This is to eliminate the possibility that teams will intentionally lose games so that more than 4 games are required to determine the World Series winner. In any given World Series players make the same money playing 4, 5, 6 or 7 games.

**4.**

**0.5/1.5 points**

 |  Previous Answers

**My Notes**

An airline, believing that 3% of passengers fail to show for flights, overbooks (sells more tickets than there are seats). Suppose that for a particular flight involving a jumbo-jet with 260 seats, the airline sells 272 tickets.

**a.** What is the expected number of ticket holders that will fail to show for the flight?



**b.** What is the probability that the airline will not have enough seats for all the ticket holders who show for the flight?



(Use 3 decimal places.)