2. (a) (i) Write down Maxwell's equations for static electric and magnetic fields in the vacuum (note that you should include charge and current densities).

   (ii) How did Maxwell modify Ampère's law to account for dynamic electric fields?

(b) In a region of space in which the relative permittivity is $\epsilon_r = 2.4$ (i.e., $\epsilon = 2.4\epsilon_0$), the relative permeability is $\mu_r = 1$, (i.e., $\mu = \mu_0$), and the free current density is $\mathbf{J} = 0$, the magnetic field, $\mathbf{B}$, is given by,

$$\mathbf{B} = A \sin(kz - \omega t) \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} \right).$$

Here A, k, and ω are constants, t is time, x, y, and z are cartesian coordinates and $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors in the x and y directions respectively.

(i) Find the auxilliary field, $\mathbf{H}$.

(ii) Find the magnetisation, $\mathbf{M}$.

(iii) Find $\nabla \cdot \mathbf{B}$ and explain the physical meaning of your result.

(iv) Identify the circumstances in which $\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$.

(v) Find $\nabla \times \mathbf{B}$.

(vi) Find $\nabla \times \mathbf{E}$.

(vii) Does $\mathbf{E} = -\nabla V$ in this case?

(viii) Find $\frac{\partial \mathbf{E}}{\partial t}$.

(ix) Deduce a possible expression for the electric field, $\mathbf{E}$.

(x) Find an expression for the polarisation vector, $\mathbf{P}$.

(xi) Find the volume density of bound charge.

(xii) Find the volume density of free charge.