Question 1

An experiment was conducted to investigate the transition pressure of bismuth as a function of temperature. Listed below in Table 3.1 are the temperature (x) in degrees Centigrade and the difference in pressure (y) from 25,000 bars in hundreds of bars for 23 samples. (So if $y = 3.66$ then the pressure is $25,000 + 3.66 \times 100 = 25,366$ bars.)

Table 3.1 The temperature and pressure difference for bismuth transition.

Temperature in ^{o}C (x)	Pressure difference in 100 bars (y)
20.8	3.66
20.9	3.56
21.0	3.36
21.9	2.56
22.1	2.67
22.1	3.06
22.4	2.37
22.5	2.67
22.5	2.02
23.0	1.57
23.1	1.57
24.8	1.38
24.8	1.48
25.0	1.43
34.0	-2.69
34.0	-2.49
34.1	-2.29
42.7	-5.76
42.7	-5.56
42.7	-5.81
49.9	-8.83
50.1	-8.98
50.1	-9.08

You can assume the following calculations:

$\sum x_i = 697.2$; $\sum x_i^2 = 23653.5$; $S_{xx} = 23653.5 - 697.2^2/23 = 2519.25$.
$\sum y_i = -18.13$; $\sum y_i^2 = 445.884$; $S_{yy} = 445.884 - 18.13^2/23 = 431.59$.
$\sum x_i y_i = -1589.63$; $S_{xy} = -1589.63 + 697.2 \times 18.13/23 = -1040.05$.

a) Produce an ANOVA table to test the 'lack of fit' and establish if it is reasonable to assume that the expected pressure is a linear function of the temperature.

b) Test the hypothesis that the slope is zero.

c) Obtain a point estimate and 95% confidence interval for the expected decrease in pressure due to an increase of 10 degrees centigrade.

Question 2

A study was conducted to see how the amount of heat, which is generated when cement sets, is influenced by the composition of the cement. The response variable is the heat generated (y) in calories per gram. The explanatory variables are the percentages of cement by weight of the constituents tricalcium-silicate (x_1) and tetracalcium-alumino-ferrite (x_2) and the data for 13 cement samples are given in Table 3.2.

Table 3.2 Heat generated in calories per gram and percentages of cement by weight of the constituents tricalcium-silicate and tetracalcium-alumino-ferrite for 13 cement samples.

Heat generated (y)	Tricalcium-aluminate (x_1)	% Tetracalcium-alumino-ferrite (x_2)
78.5	26	6
74.3	29	15
104.3	56	8
87.6	31	8
95.9	52	6
109.2	55	9
102.7	71	17
72.5	31	22
93.1	54	18
115.9	47	4
83.8	40	23
113.3	66	9
109.4	68	8

A multiple regression model was fitted to these data with heat generated as a linear function of x_1 and x_2 with an intercept term using MINITAB.

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Sum of y = 1240.5

Sum of x1 = 626

Sum of x2 = 153

Regression Analysis: y versus x1; x2

The regression equation is
y = **.** + 0.731 x1 - 1.01 x2
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Predictor      Coef  SE Coef      T      P
Constant     **.***    7.383   *.**  0.000
x1           0.7313   0.1207   *.**  0.000
x2          -1.0084   0.2934  **.**  0.006

S = 6.44548   R-Sq = **.*%   R-Sq(adj) = 81.6%

Analysis of Variance

Source          DF      SS      MS      F      P
Regression       *  ****.*  ****.*  **.**  *.***
Residual Error  **   ***.*    **.*
Total           **  ****.*

Source  DF  Seq SS
x1       1  1809.4
x2       1   490.9
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a) Obtain the ANOVA table for the model with predictors x_1 and x_2.

b) Which percentage of the variability of y is explained by the overall model?

c) Test the hypothesis that $\beta_1 = \beta_2 = 0$. What do you conclude?

d) Compare the model with two linear predictors to the one with a single predictor x_1 and decide which one is the best.

e) Using the Minitab listing, calculate $\hat{\beta}_0$.

f) Test the hypothesis $\beta_i = 0$ for each $i = 0, 1, 2$ individually. What do you conclude?

g) Calculate the 95% confidence intervals for $\beta_0, \beta_1, \beta_2$.